Deterministic Margin Selection Tool for Single Iso Multi-Target Radiosurgery

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ABSTRACT

Purpose
We introduce a deterministic, analytic framework to compute minimal geometric margins for single-isocenter, multi-target radiosurgery that guarantees a user-specified maximum loss of target coverage. The method links target size, allowable rigid rotation about the isocenter, and distance from isocenter to an explicit coverage-loss metric (for example tumor V100 = 95%), producing a closed-form margin estimation for each met.

Methods
Assuming rigid-body rotation about a fixed isocenter, we derive expressions for the maximal surface displacement of a spherical target as a function of its diameter and off-axis distance. The overlap of this displaced target with a spherical dose distribution at its expected location can be mathematically computed. By varying the margin, the size of the dose distribution sphere can be adjusted such that there is a maximum overlap for a given acceptable rotation. The formulation accepts a maximum rotation angle representing immobilization and setup uncertainty and yields margins as an explicit function of target diameter and distance from isocenter. We validate analytic predictions for target diameters spanning 2–30 mm, distance from isocenter 0–75 mm, and rotation tolerances up to 2°.

Results
Margins scale nonlinearly with radial distance and target size. Smaller peripheral targets demand substantially larger margins than centrally located lesions. With a 2° rotational tolerance and a required V100 of 95%, a 5-mm tumor requires margins of 0.7 mm and 2.2 mm at 30 mm and 75 mm from isocenter, respectively, while a 20-mm tumor under the same conditions requires corresponding margins of 0.3 mm and 1.4 mm. The model identifies parameters where single-isocenter strategies remain clinically acceptable and where multiple isocenters are preferable.

Conclusions
The instantaneous margin calculation enables lesion-specific margins within a single isocenter, providing a deterministic and clinically actionable method for objective margin selection in multi-target radiosurgery.

INTRODUCTION

Single-isocenter multi-target (SIMT) radiosurgery enhances efficiency by treating multiple lesions with a single plan. However, this technique is highly susceptible to patient positioning errors, particularly rotational deviations. Even minor rotations can lead to significant reductions in target dose coverage, with the degree of degradation being influenced by target size, the magnitude of rotation, and the target's distance from the treatment isocenter.

We developed mathematical models to simulate the dosimetric consequences of rotational errors under the assumption of a dose distribution encompassing the target perfectly, in the absence of setup errors. The primary objective is to elucidate the interplay between target size, distance from the isocenter, and the magnitude of rotational error, with a view to informing individualized planning target volume (PTV) margin selection for optimized target coverage and minimized normal tissue irradiation.

METHODS AND MATERIALS

Mathematical Modeling of Target and Radiation Spheres

The analysis assumes a Conformity Index (CI) = 1.0, such that the radiation perfectly encompasses the target in an ideal scenario, where CI is defined as

$$CI = \frac{\text{Volume covered by the prescription dose}}{\text{Volume of the target}}$$

The equation of a radiation sphere centered at distance s from isocenter can be represented as

$$(x-s)^2 + y^2 + z^2 = {r_1}^2$$

The equation of the target sphere also centered at s but rotated by an angle t can be represented as

$$(x - s*cos(t))^2 + (y - s*sin(t))^2 + z^2 = r_2^2$$

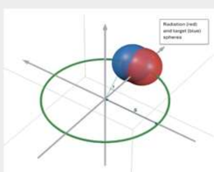


Figure 1 shows an illustration of target and radiation spheres at a distance s from isocenter and rotated by an angle t°. The isocenter is the origin of the coordinate system.

The above formula can be modified to incorporate rotations in two dimensions (t and a) with the new equation for target sphere represented as $(x - s*cos(t)\cos(a))^2 + (y - s*sin(t))^2 + (z - s*\cos(t)\sin(a))^2 = r_2^2$

If d is the distance between the centers of the two spheres, the volume of overlap can be derived[3] as

$$V = \frac{\pi*(r_1^2 + {r_2}^2 - d)^2 * \left[d^2 + 2d({r_1}^2 + {r_2}^2) - 3({r_1}^2 - {r_2}^2)^2\right]}{12d}$$

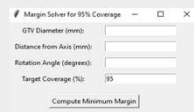
Once the volume of overlap is known, the percent overlap with radiation volume can be determined

Using the above set of formulas, we can derive the volume and thus precent of overlap between target and radiation spheres, for various target sizes r_2, angle of rotation t, and distance from isocenter s. The size of the radiation sphere can be modeled as a safety margin (m) on the target sphere, such that $r_1 = r_2 + m$. A software program to be used by physicians was created to generate this analysis

Margin Selection

The problem can be inverted to select optimal margins based on the expected coverage.

The value of margin m can be varied, and the Volume of overlap V can be computed at each step. This allows for individualized margin selection for each tumor based on its size, location rotational tolerance and desired coverage.



Screenshot of the margin solver software program to aid radiation oncologists in their treatment decisions

RESULTS

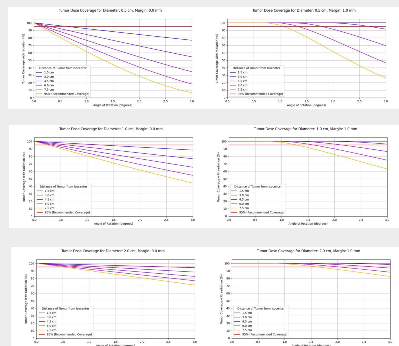


Figure 2 shows percent of tumor covered by radiation for differently sized tumors as a function of distance from isocenter and degree of rotation. The graphs on the left show the coverage in the absence of any safety margins and the right sided graphs shows the coverage with 1mm margin

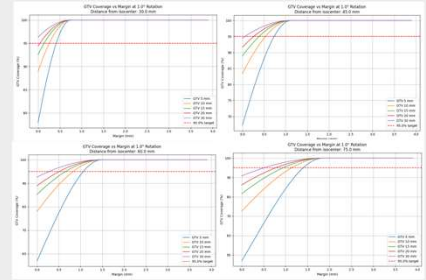


Figure 3 shows the margins required various sized mets at fixed distances from the isocenter. Smaller tumors require larger margins as expected for the same distance from isocenter. As the distance from isocenter increases, the amount of margin required also increases across the board

GTV Sizes (mm)	Distance of GTV from Isocenter (cm)				
	1.5	3.0	4.5	6.0	7.5
5	0.2	0.5	0.8	1.1	1.5
10	0.1	0.3	0.6	0.9	1.2
15	0	0.2	0.4	0.7	1.0
20	0	0.1	0.3	0.6	0.8
30	0	0	0.1	0.3	0.6

Table 1 shows the minimum margins in mm required to achieve a GTV V100 > 95% as a function of GTV sizes and distance from isocenter assuming a one-degree rotation threshold

DISCUSSION

The mathematical analysis show that treatment margins are important, however, the size of margin can be varied as a function of tumor size and location.

Smaller mets are more susceptible to coverage loss as both distance from isocenter and degree of rotation increases.

A uniform margin of 1mm may not be sufficient for a small met far away from the isocenter. Conversely, a 1mm margin maybe an over kill for a large met that is located close to the isocenter.

CONCLUSIONS

The instantaneous margin calculation enables lesion-specific margins within a single isocenter, providing a deterministic and clinically actionable method for objective margin selection in multi-target radiosurgery.

The required margin is a function of the met size and distance from isocenter, assuming rotations above a certain threshold can be corrected.

The graphs and tables presented can be used to determine scenarios where a single isocenter may not be the most clinically suitable option and a multiple isocenter plan maybe the prudent choice for radiosurgery

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