

INTRODUCTION

Hepatic Artery Vasculature

- Liver Y-90 radioembolization planning can be done with patient-specific hemodynamic modeling in the hepatic artery through computational fluid dynamics (CFD)
- Conventional mesh-based CFD is too slow and geometry-sensitive for clinical adoption
- Motivates the development of mesh-free **physics-informed neural networks (PINNs)** to resolve the full pulsatile cardiac cycle.

$$Q_{Inlet} = \text{Time-Dependent Flow Rate (1X)}$$

$$P_{Outlet} = \text{Windkessel BC (46X)}$$

$$\rho_{Blood} = 1060.0 \frac{kg}{m^3}$$

$$\mu_{Blood} = 4.00 \text{ mPa}\cdot\text{s}$$

$$Re = \rho \cdot U_{avg} \cdot D / \mu$$

$$\approx 251 \text{ (laminar)}$$

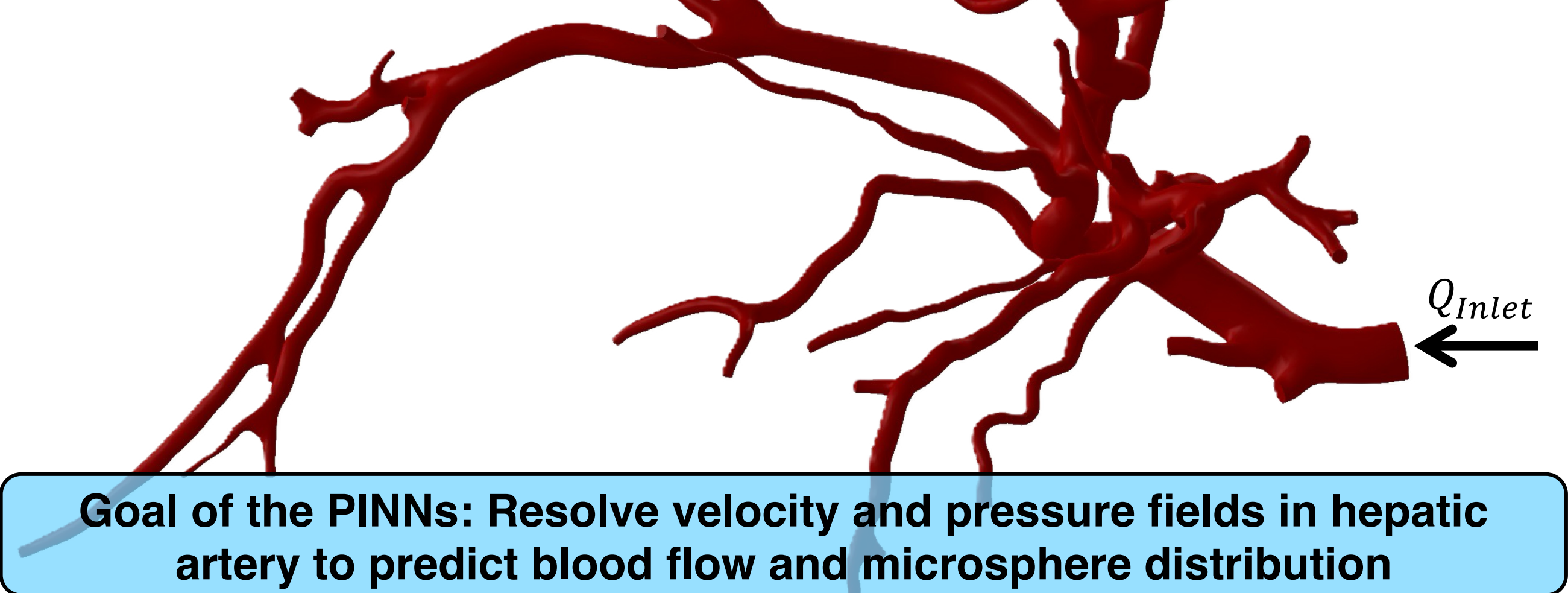


Fig. 1. Patient-specific hepatic arterial tree from contrast CT — one inlet, 46 outlets across segments S5-S8 and tumor.

Why Physics-Informed Neural Networks?

Physics-Informed Neural Networks (PINNs) embed the governing PDEs directly into the loss function, letting a single mesh-free network approximate velocity and pressure fields from a sparse set of reference samples.

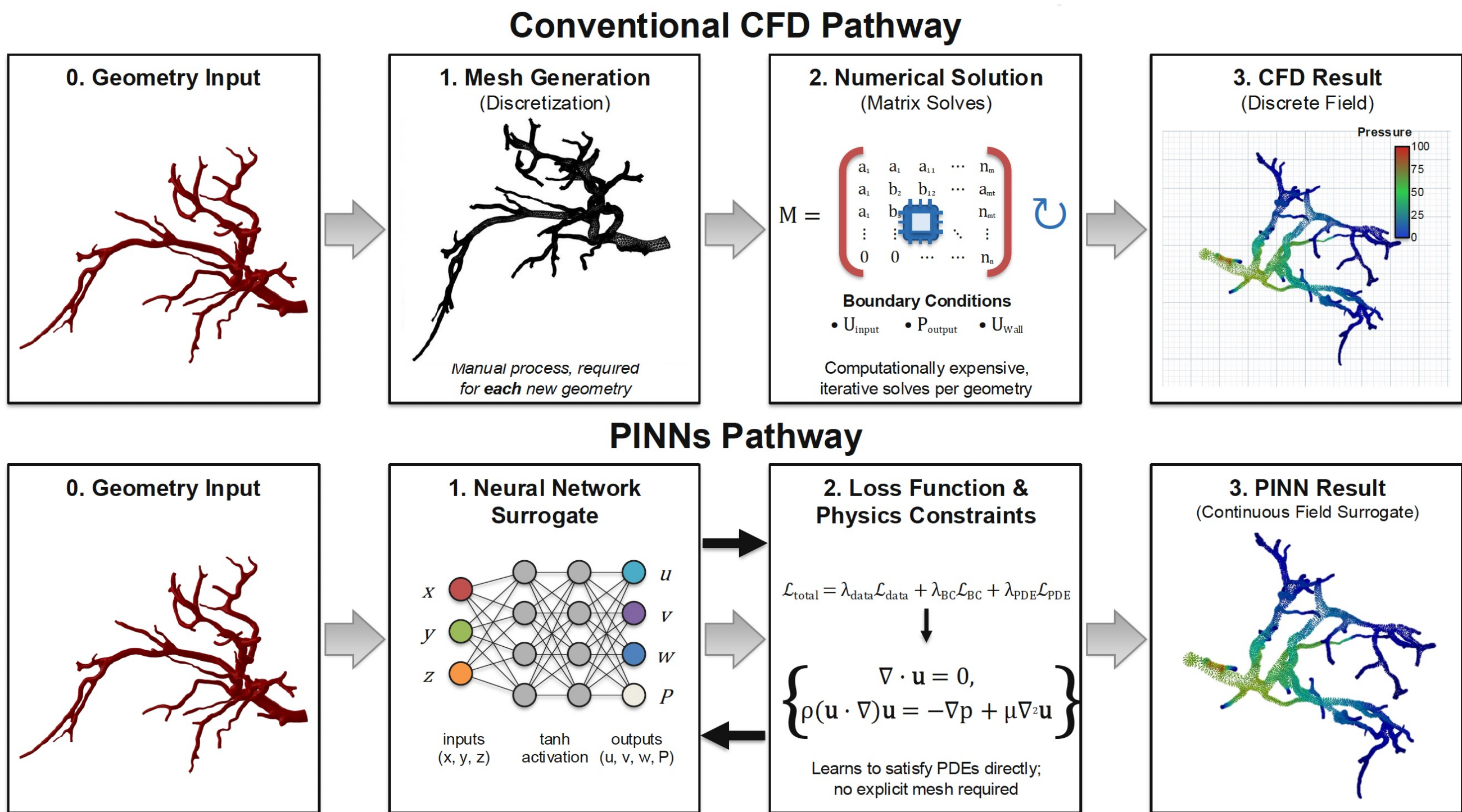


Fig. 2. Conventional CFD vs. PINNs: CFD needs mesh generation and matrix solves per geometry; PINNs provide a continuous field surrogate.

Scope of This Study

Key advantages for this problem:

- Mesh-free evaluation on complex vessel geometry
- Continuous, differentiable surrogate of u, v, w, p
- Hybrid PDE + data loss reduces required training samples

In this work we train a 3D, transient, incompressible PINNs for a patient-specific hepatic artery tree using Nvidia PhysicsNeMo and validate against a (CFD) COMSOL finite-element solution on the same geometry.

Aims: We combine:

- Unsteady nondimensional Navier–Stokes residual loss via automatic differentiation
- Pulsatile Fourier inlet, 46 Windkessel outlets, and no-slip walls
- Validation vs. a CFD transient solution; Y-90 dose from per-outlet flow

PROPOSED MODEL

Governing Equations – Unsteady Navier–Stokes

Continuity (mass conservation) and $x/y/z$ momentum equations are written in nondimensional form with the characteristic diameter D , inlet mean velocity U_{avg} , and Reynolds number Re . CFD = ground truth to compute these parameters, done in COMSOL Multiphysics software.

Governing PDEs (Nondimensional, Constant Mass Density):

$$\nabla \cdot \mathbf{u}^* = 0 \quad (\text{Continuity})$$

$$\partial \mathbf{u}^* / \partial t^* + (\mathbf{u}^* \cdot \nabla) \mathbf{u}^* = -\nabla p^* + (1/Re) \nabla^2 \mathbf{u}^* \quad (\text{Momentum})$$

Nondimensionalization:

$$x^* = x/D, u^* = u/U_{avg}, p^* = p/(\rho U_{avg}^2), Re = (\rho U_{avg} D)/\mu$$

Residuals (Automatic Differentiation): r_{cont}, r_x, r_y, r_z

Boundary Conditions: Inlet (Mass Flow Rate), Outlet (Windkessel), Walls (No-slips)

PINNs Architecture & Training

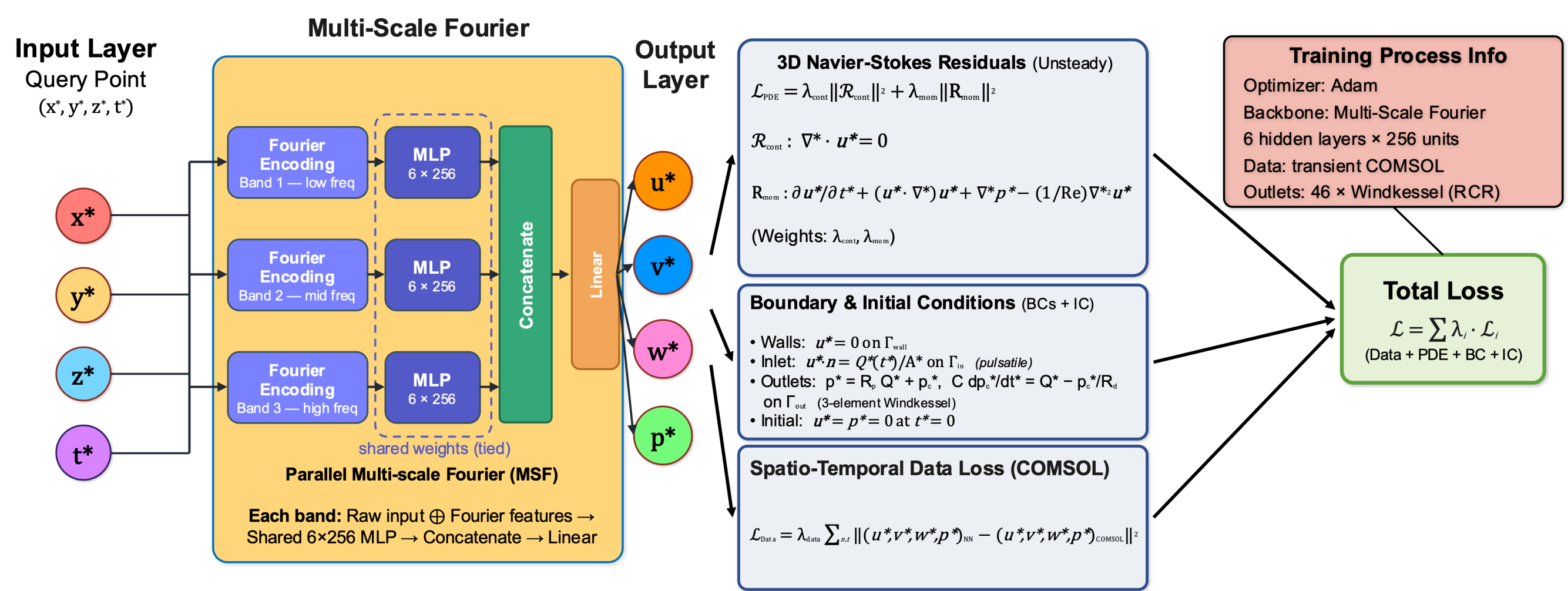


Fig. 3. PINNs architecture: a fully-connected network $N_\theta: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ maps nondimensional coordinates (x^*, y^*, z^*, t^*) to (u^*, v^*, w^*, p^*) . 6×256 hidden units, SiLU activation, trained with Adam for $\sim 858K$ steps.

Training loss: $L_{total} = \lambda_{PDE} L_{PDE} + \lambda_{BC} L_{BC} + \lambda_{IC} L_{IC} + \lambda_{data} L_{data}$
PDE residual: The average mean of $|r_{cont}|^2 + |r_x|^2 + |r_y|^2 + |r_z|^2$
Exponential Learning-rate schedule: $lr(t) = 10^{-3} \times 0.95^{t/20000}$
Outputs: u^*, v^*, w^*, p^* are the nondimensional velocity components and pressure predicted by the network at query point (x^*, y^*, z^*, t^*) . Training uses Adam for $\sim 858K$ steps on COMSOL samples.

RESULTS

PINNs vs. COMSOL — Velocity & Pressure Fields

PINNs predictions match COMSOL across the full 3D hepatic-artery lumen for both velocity magnitude and pressure. Point-wise error is well below 1% of the field range (pressure median 0.04%, velocity median 0.21%), and parity plots show strong linear agreement ($R^2 \geq 0.89$ for all components, 0.999 for pressure). Residual errors concentrate near sharp bifurcations, consistent with the highest local gradients.

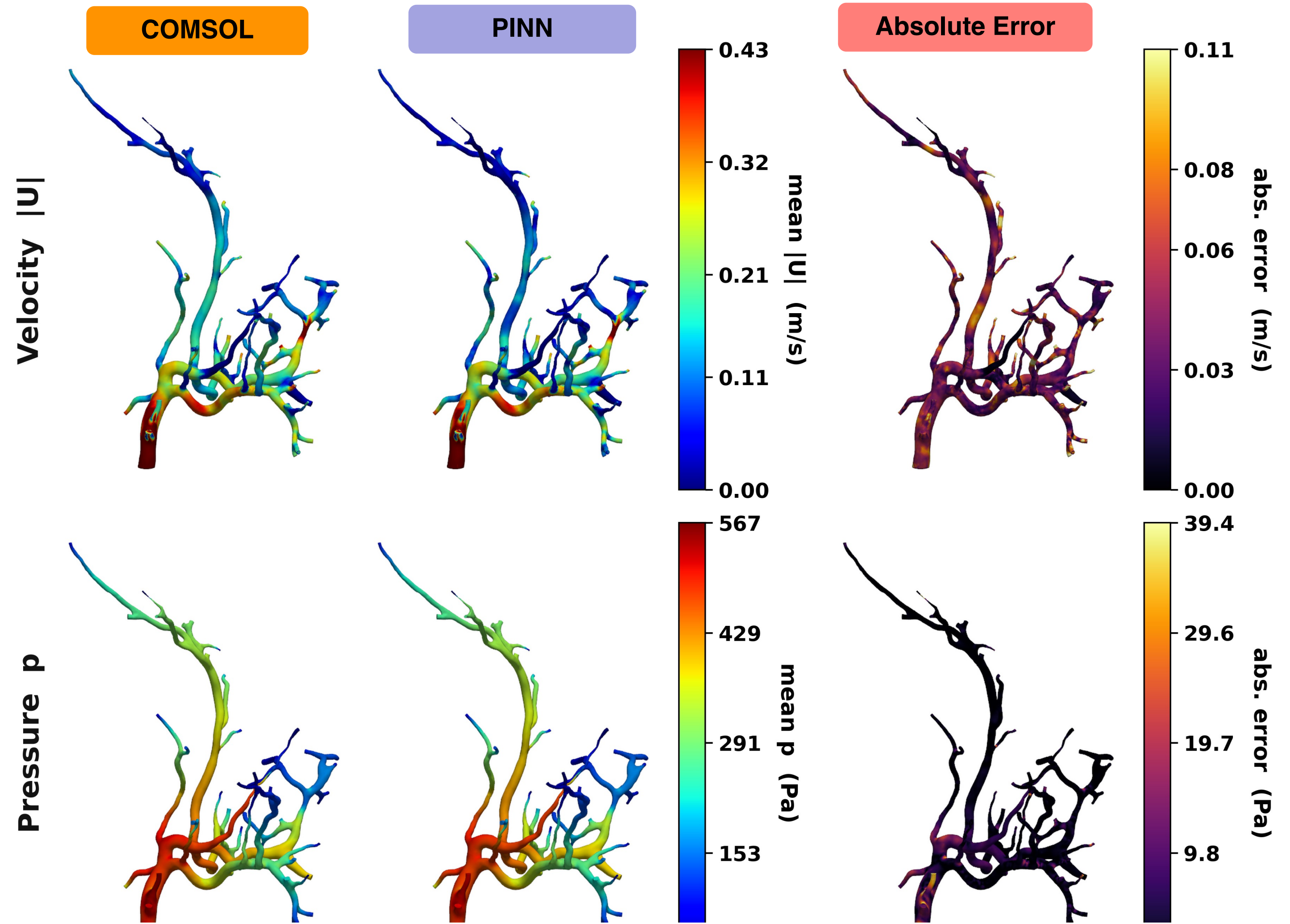


Fig. 4. Cycle-averaged velocity U^* (top) and pressure (bottom): COMSOL vs. PINNs vs. absolute error.

The prescribed Fourier inlet waveform is recovered to 8.4% relative error, and the domain-mean velocity tracks COMSOL through systole and diastole. The 46 Windkessel (RC) outlets distribute flow across all branches — the cycle-mean split matches COMSOL to 26% L^1 error.

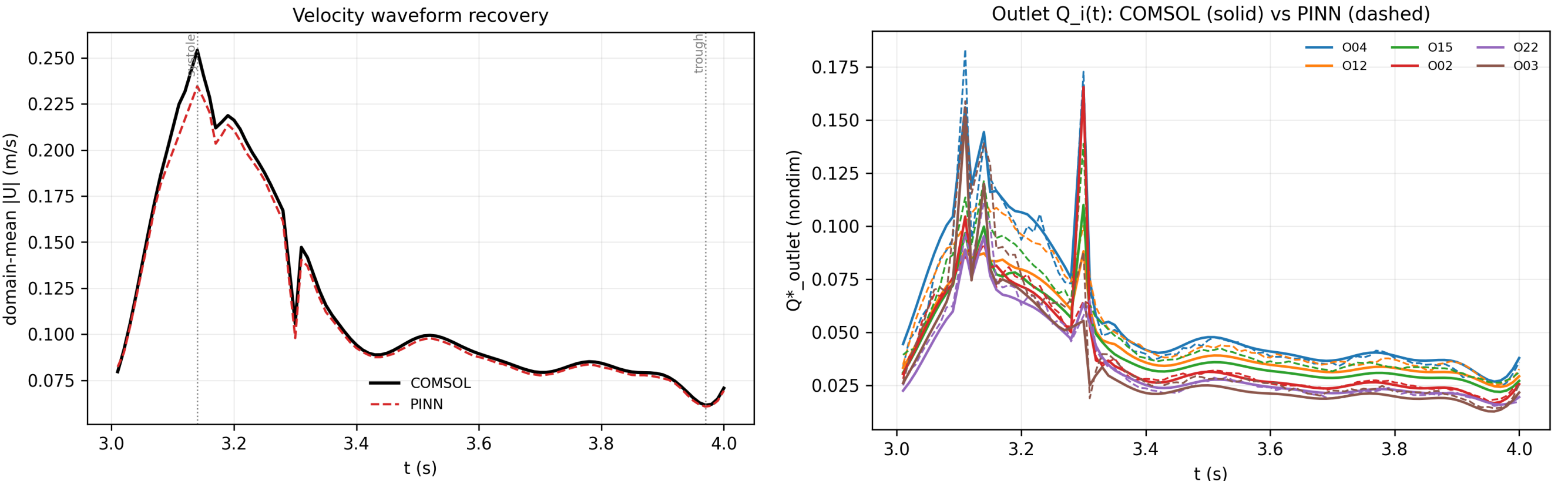


Fig. 5. Domain-mean velocity waveform recovery and per-outlet flow $Q(t)$ — COMSOL (solid) vs. PINN (dashed).

Accuracy Across the Cardiac Cycle

- COMSOL provides reference u^*, v^*, w^*, p^* ; the PINNs matches these while minimizing the Navier-Stokes residual.
- Over $\sim 858K$ Adam steps, total loss and PDE residuals each decay ~ 5 decades.
- Pooled relative L_2 vs. COMSOL: u^* 16.0%, v^* 29.9%, w^* 26.0%, p^* 1.7%.
- Inlet monitors track the prescribed pulsatile waveform (peak 2.17 & trough 0.67); wall $|U^*| \leq 0.13$ confirms BC enforcement.
- Per-timestep R^2 stays near unity, dipping only at peak systole where velocity gradients are steepest.

Fig. 6. Pooled PINN-vs-COMSOL parity over all timesteps: $R^2 = 0.97/0.92/0.95/0.998$ for $u^*/v^*/w^*/p^*$.

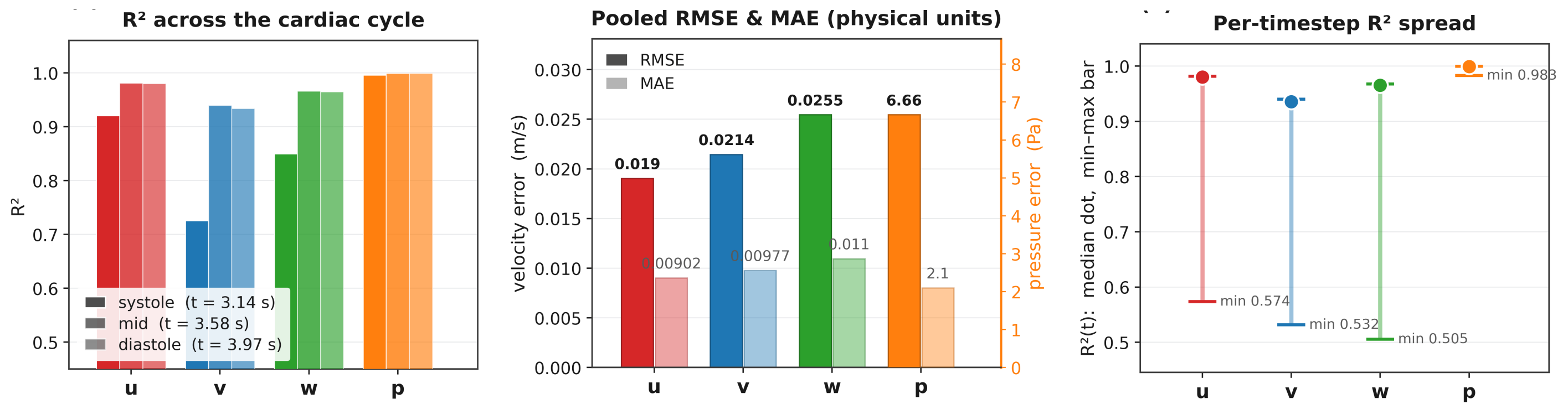
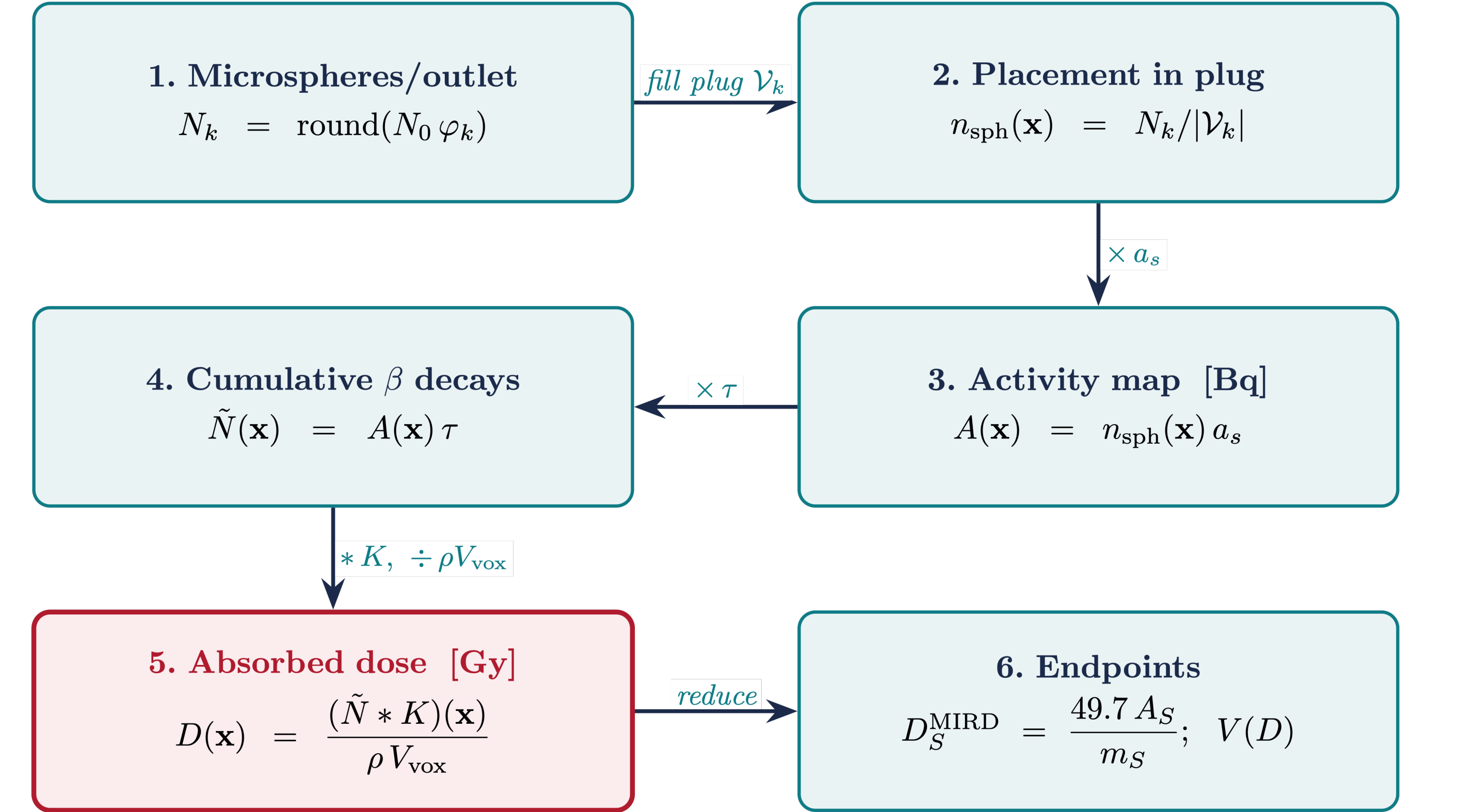


Fig. 7. Per-field accuracy vs. CFD over 218,437 mesh points $\times$ 97 clean timesteps: R^2 at systole/mid/diastole, pooled RMSE & MAE in physical units, and per-timestep R^2 spread.

From Flow to Y-90 Dose - Predicted Dose Distribution & DVH

Cumulative per-outlet flow sets each branch’s microsphere share; spheres follow streamlines and a Y-90 dose-point kernel gives the 3-D dose map (Gy).



- ϕ_k = outlet flow fraction, V_k = voxel set of plug k , $*$ = 3-D convolution;
- K Y-90 β dosepoint kernel (GATE Monte Carlo).

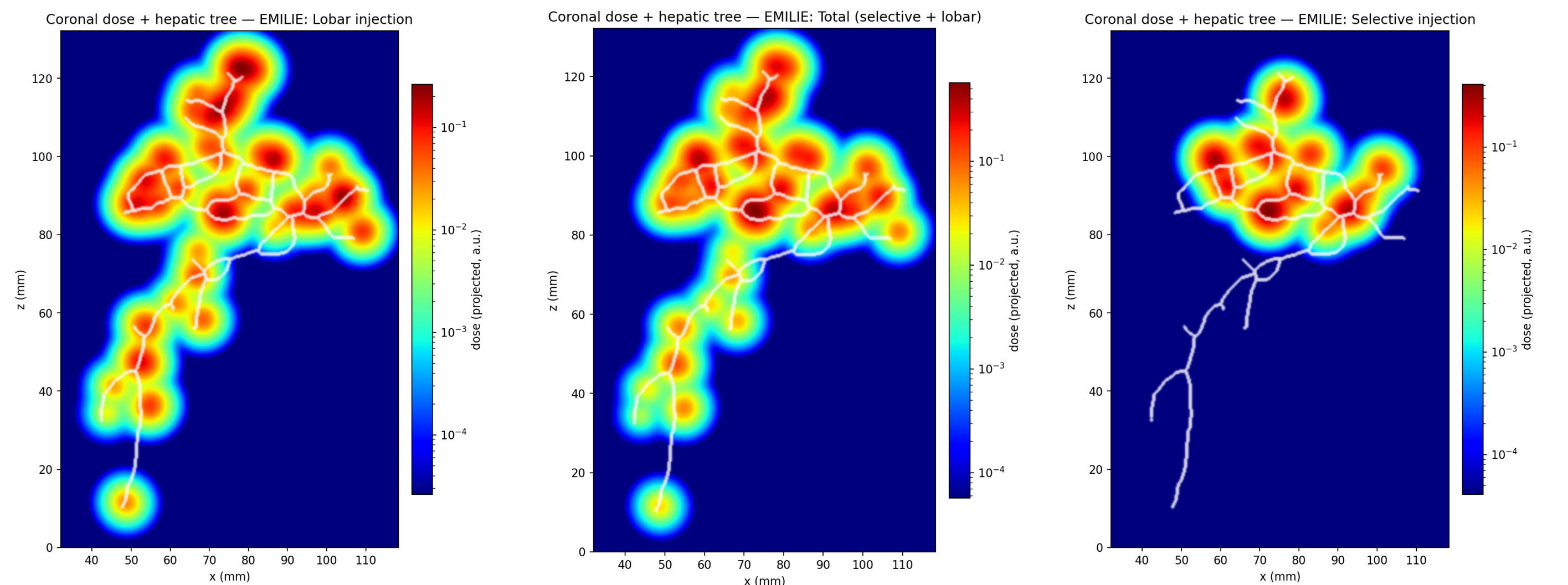


Fig. 8. Y-90 coronal dose over the hepatic arterial tree — lobar, combined (selective + lobar), and selective injection.

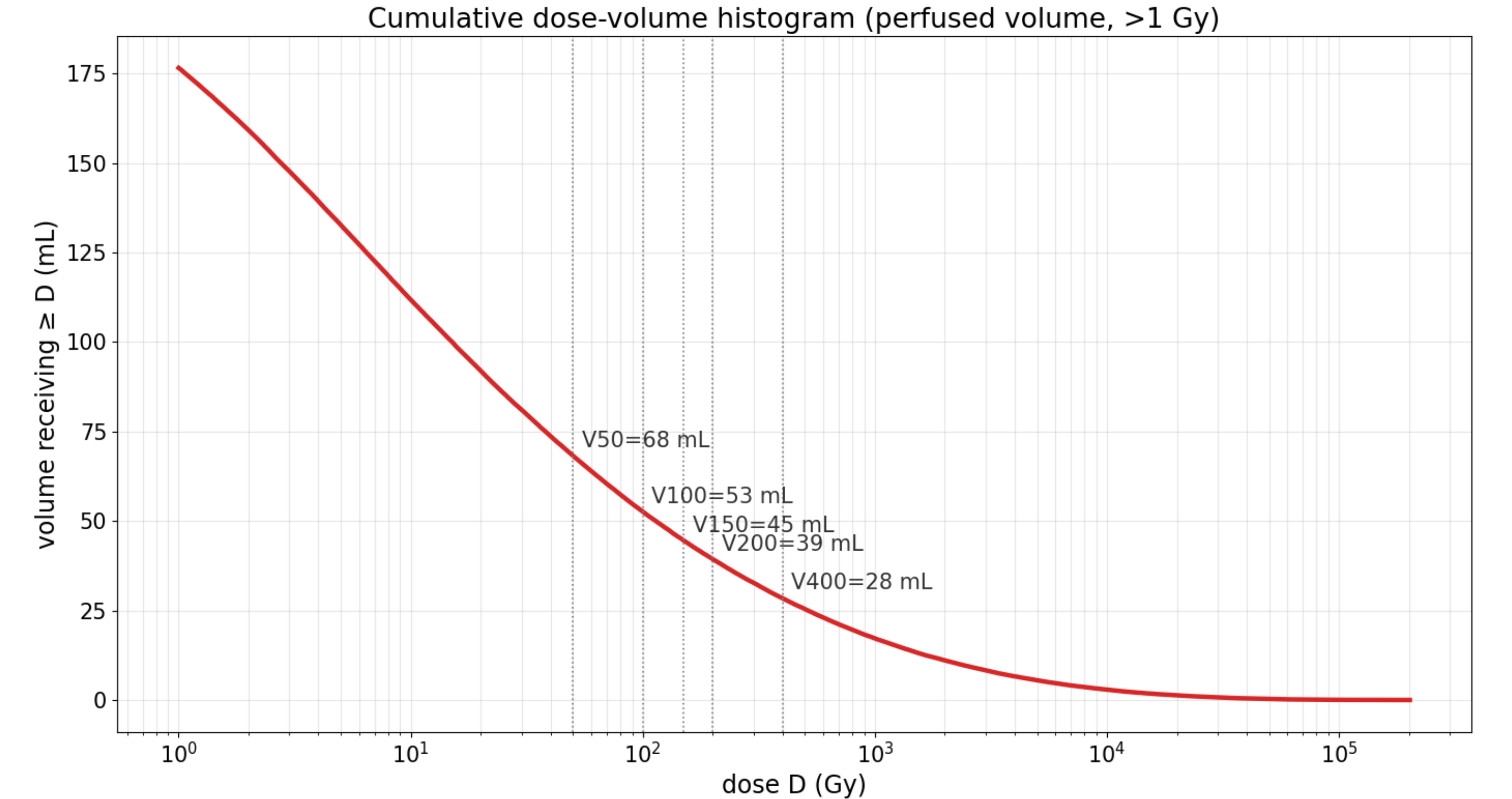


Fig. 9. Cumulative dose–volume histogram (perfused volume > 1 Gy, 177 mL): $V_{50} / V_{100} / V_{200} = 68 / 53 / 39$ mL.

CONCLUSIONS

- Transient PINNs reproduces the cardiac-cycle hepatic flow field ($R^2 = 0.97 / 0.89 / 0.93 / 0.999$ for $u / v / w / p$; inlet flow within 8.4%, full COMSOL mesh).
- Y-90 dose is robust to flow error; both PINNs and COMSOL dose agree within ~ 1 –2% despite a 26% per-outlet flow-split difference.
- Peak-systole jet cores are the accuracy limit; the mesh-free surrogate otherwise generalizes across the lumen.

REFERENCES

- [1] Talaat, M., Si, X., Dong, H., & Xi, J. (2025). Physics-informed neural networks simulation and validation of airflows in three-dimensional upper respiratory tracts. *Fluids*, 10(12), 306.
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- [3] Roncali, E., Taebi, A., Foster, C., & Vu, C. T. (2020). Personalized dosimetry for liver cancer Y-90 radioembolization using computational fluid dynamics and Monte Carlo simulation. *Annals of biomedical engineering*, 48(5), 1499–1510.