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Monte Carlo simulation of the diffusion-leakage model in Alpha DaRT: Methodological pitfalls and acceleration shortcuts

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INTRODUCTION

In diffusing alpha-emitters radiation therapy (Alpha DaRT), unsealed sources are implanted in solid tumors and release radionuclides [1]. The diffusion-leakage (DL) model [1] is used to calculate the spread of the released radionuclides and their decay products. Four physical processes are included in the model: radioactive decay, desorption from the source, diffusion in the tumor, and leakage from the tumor (see Figure 1).

Various numerical methods for simulating the DL model have been published [2,3,4,5,6]. Until recently, all of them were grid-based time-marching deterministic methods [2,3,4]. A stochastic method using the Geant4 Monte Carlo (MC) toolkit was published by Ballisat *et al* in 2024 [5] but was incorrectly calculating radionuclides radial displacement (Δr) due to diffusion. Another method, still using Geant4, was published by Lielkajis *et al* in 2026 [6], this time correctly calculating radionuclides displacement but in Cartesian coordinates (Δx, Δy, Δz).

This work summarizes the probability distributions to use to develop basic and accelerated stand-alone grid-free MC methods. It validates the method by simulating a point source in a homogeneous medium and comparing the resulting decay positions distribution to analytical expressions. It also highlights two pitfalls to avoid and analytically evaluate their impact on radionuclides radial displacement.

METHODS

1) The basic stand-alone grid-free MC method

The following random samplings are used to simulate individually each of the four physical processes of the DL model. They allow to navigate in the diagram of Figure 1, following a single atom from its initial state of ^{224}Ra radionuclide on the source to its last state of stable ^{208}Pb. Three-dimensional (3D) displacements in the tumor due to the diffusion process can be calculated either in Cartesian or spherical coordinates.

Radioactive decay:

radionuclide lifetime: $t_{\lambda,\mathrm{nuc}} \sim \mathrm{Exp}(\lambda = \lambda_{\mathrm{nuc}} = \ln(2)/t_{12,\mathrm{nuc}})$

^{212}Bi branching: $N_{\mathrm{Br,BiTl}} \sim \mathrm{Bernoulli}\left(p = P_{\mathrm{Br}}(\mathrm{Bi} \to \mathrm{Tl})\right)$

Desorption:

^{220}Rn: $N_{\mathrm{des,Rn}} \sim \mathrm{Bernoulli}\left(p = P_{\mathrm{des}}(\mathrm{Rn} \leftarrow \mathrm{Ra})\right)$

^{212}Pb: $N_{\mathrm{des,Pb}} \sim \mathrm{Bernoulli}\left(p = P_{\mathrm{des}}^{\mathrm{eqv}}(\mathrm{Pb} \leftarrow \mathrm{Po} \leftarrow \mathrm{Rn})\right)$

Diffusion:

radionuclide 3D displacement:

$$\begin{aligned} \Delta x &\sim \mathcal{N}(\mu = 0, \sigma^2 = 2D_{\mathrm{nuc}}\, t_{\lambda,\mathrm{nuc}}) \\ \Delta y &\sim \mathcal{N}(\mu = 0, \sigma^2 = 2D_{\mathrm{nuc}}\, t_{\lambda,\mathrm{nuc}}) \\ \Delta z &\sim \mathcal{N}(\mu = 0, \sigma^2 = 2D_{\mathrm{nuc}}\, t_{\lambda,\mathrm{nuc}}) \end{aligned} \quad \Bigg| \quad \begin{aligned} \Delta r &\sim \mathrm{Maxwell}\left(a = \sqrt{2D_{\mathrm{nuc}}\, t_{\lambda,\mathrm{nuc}}}\right) \\ \theta &\sim \mathcal{U}(a = 0, b = \pi) \\ \phi &\sim \mathcal{U}(a = 0, b = 2\pi) \end{aligned}$$

Leakage:

time to leakage: $t_{\alpha,\mathrm{nuc}} \sim \mathrm{Exp}(\lambda = \alpha_{\mathrm{nuc}})$

NOTE: Half-lives of radionuclides ($t_{12,\mathrm{nuc}}$) and branching ratios (P_{Br}) are physical constants (see www.nndc.bnl.gov/nudat3/). Desorption probabilities (P_{des} and $P_{\mathrm{des}}^{\mathrm{eqv}}$) are source properties from manufacturer. Effective diffusion coefficients (D_{nuc}) and leakage rate constants (α_{nuc}) are tumor properties.

METHODS (Continued)

2) Shortcuts to accelerate the basic method

1. Categorizing atoms according to their path from ^{224}Ra to ^{208}Pb allows to focus on those undergoing decays in the tumor (see Figure 2).
2. For permanent treatments, when the interest is only in decay positions, expected displacements are obtained directly, without sampling first for the diffusion time.

$$\begin{aligned} |\Delta k| &\sim \mathrm{Exp}(\lambda = 1/L_{\mathrm{nuc}}^{\infty}) \\ N_{\mathrm{sgn},\Delta k} &\sim \mathrm{Bernoulli}(p = 0.5) \\ \Delta k &= (-1)^{N\mathrm{sgn},\Delta k} \cdot |\Delta k| \end{aligned} \quad \Bigg| \quad \begin{aligned} \Delta r &\sim \mathrm{RadDisp}(a = L_{\mathrm{nuc}}^{\infty}) \\ \theta &\sim \mathcal{U}(a = 0, b = \pi) \\ \phi &\sim \mathcal{U}(a = 0, b = 2\pi) \end{aligned}$$

Note: In equations above, $k = \{x, y, z\}$, $L_{\mathrm{nuc}}^{\infty} \equiv \left[\frac{p_{\mathrm{nuc}}}{(\lambda_{\mathrm{nuc}}+\alpha_{\mathrm{nuc}})}\right]^{1/2}$, and the PDF for the "RadDisp" distribution is given by: $\mathrm{PDF}_{\mathrm{RadDisp}}(r; a) = \left(\frac{1}{a}\right)(r/a)\, e^{-(r/a)}$

3) Pitfalls to avoid (incorrect samplings)

1. For temporary treatments or when interested in decay times, normal distributions are associated with radionuclides Cartesian displacements in the tumor. It is incorrect to also use a normal distribution to obtain radionuclides radial displacement (error made by Ballisat *et al* 2024 [5]). The correct one is the Maxwell-Boltzmann distribution (see random samplings in the basic method).
2. The lifetime distribution of radionuclides decaying specifically in the tumor is affected by the leakage process (i.e. radionuclides with a longer lifetime have more chances to leak). It is incorrect to not take this into account.

Incorrect: $(t_{\lambda,\mathrm{nuc}})_{\mathrm{tum}} \sim \mathrm{Exp}(\lambda = \lambda_{\mathrm{nuc}})$

Correct: $(t_{\lambda,\mathrm{nuc}})_{\mathrm{tum}} \sim \mathrm{Exp}(\lambda = (\lambda_{\mathrm{nuc}} + \alpha_{\mathrm{nuc}}))$

The impact of pitfalls on radionuclides radial displacement is shown in Figure 4.

4) Validation

- An exact analytical expression is available for the volumetric PDF (vPDF) of the expected displacement of radionuclides leaving a point source, diffusing in a homogeneous medium, and being subject to a uniform leakage:

$$\mathrm{vPDF}_{\mathrm{RadDisp}}(r; a = L_{\mathrm{nuc}}^{\infty}) = \frac{\mathrm{PDF}_{\mathrm{RadDisp}}(r; a = L_{\mathrm{nuc}}^{\infty})}{4\pi r^2} = \frac{1}{4\pi (L_{\mathrm{nuc}}^{\infty})^3} \frac{e^{-(r/L_{\mathrm{nuc}}^{\infty})}}{(r/L_{\mathrm{nuc}}^{\infty})}$$

Since voxels are necessary to score decay events, this expression needs to be voxel-averaged. For spherical voxels extending from radius r_a to r_b:

$$\overline{\mathrm{vPDF}_{\mathrm{RadDisp}}}(r_a \le r < r_b; a = L_{\mathrm{nuc}}^{\infty}) = \frac{\left(1+\frac{r_a}{L_{\mathrm{nuc}}^{\infty}}\right) e^{-(r_a/L_{\mathrm{nuc}}^{\infty})} - \left(1+\frac{r_b}{L_{\mathrm{nuc}}^{\infty}}\right) e^{-(r_b/L_{\mathrm{nuc}}^{\infty})}}{\frac{4\pi}{3}(r_b^3 - r_a^3)}$$

These expressions are used as a reference for validation in Figure 3 and Figure 4.

- Since the developed method is stochastic, a proper validation must include a fluctuation envelope around the reference curve. This envelope is calculated voxel by voxel using a binomial distribution for which the number of trials is the number of ^{224}Ra radionuclides on the source at the beginning of the treatment and for which the mean is the value of the reference curve. This envelope is displayed in Figure 3.

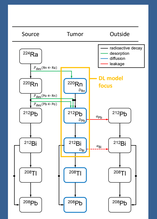

Figure 1. Physical processes in the DL model.

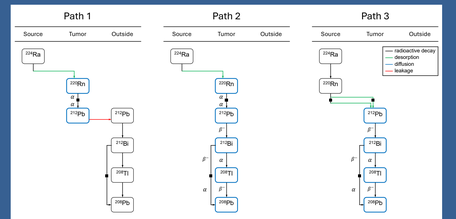

Figure 2. Paths of the DL model leading to at least one alpha decay in the tumor.

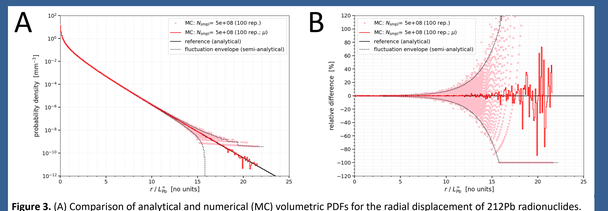

Figure 3. (A) Comparison of analytical and numerical (MC) volumetric PDFs for the radial displacement of 212Pb radionuclides. (B) Relative difference between analytical and numerical results.

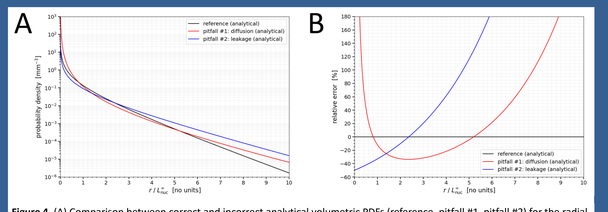

Figure 4. (A) Comparison between correct and incorrect analytical volumetric PDFs (reference, pitfall #1, pitfall #2) for the radial displacement of 212Pb radionuclides. (B) Relative difference between analytical results.

RESULTS

Validation

- As a validation of the developed MC method, Figure 3 compares analytical and numerical distributions of the radial displacement of ^{212}Pb radionuclides leaving a point source and diffusing in a homogeneous tumor while being subject to a uniform leakage. 5×10^8 ^{224}Ra radionuclides were placed on the source and the simulation was repeated 100 times. Hence, the solid red curve corresponds to the mean of 5×10^{10} ^{224}Ra radionuclides initially on the source (approximately 3 μCi). The width of the fluctuation envelope around the reference curve is 95 %. Figure 3(B) shows that the numerical mean curve correctly follows the reference curve.

Impact of pitfalls on radionuclides radial displacement

- Figure 4 shows that an incorrect simulation of the diffusion process (pitfall #1) leads to an overestimation of the number of decays near and far from the source and an underestimation (up to 35%) at distances between 0.8 and 5.2 diffusion lengths from the source. Also, an incorrect simulation of the leakage process (pitfall #2) leads to an underestimation (up to 50%) of the number of decays near the source and an overestimation far from the source (> 2.4 diffusion lengths).

DISCUSSION

- Limitations: Developed MC methods provide benchmark solutions for point and line sources in a homogeneous isotropic medium. Handling sources with finite diameters is discussed by Lielkajis *et al* 2026 [6].
- Advantages: Developed MC methods are grid-free, easy to implement, capture the inherent stochasticity of the modeled physical processes and directly provide the associated uncertainty when repeating a simulation multiple times.
- Disadvantages: Developed MC methods are computationally expensive compared to grid-based deterministic methods, especially if variance reduction methods are not applied.

CONCLUSIONS

It is now possible for anyone interested in Alpha DaRT to easily develop its own stand-alone grid-free MC code simulating the DL model.

REFERENCES

1. Arazi, L. (2020). Diffusing alpha-emitters radiation therapy: approximate modeling of the macroscopic alpha particle dose of a point source. Physics in Medicine & Biology, 65(1), 015015.
2. Heger, G., Roy, A., Dumancic, M., & Arazi, L. (2023). Alpha dose modeling in diffusing alpha-emitters radiation therapy-Part I: single-seed calculations in one and two dimensions. Med Phys, 50(3), 1793-1811.
3. Zhang, I. P., Cohen, G. N., & Damato, A. L. (2024). A finite element method for modeling diffusion of alpha-emitting particles in tissue. Med Phys, 51(3), 2263-2276.
4. Chevé, P., Charbonneau, P., Carrier, J.-F., & Kamio, Y. (2024). A finite volume approach to dosimetric calculations in diffusing alpha-emitters radiation therapy. Phys Med Biol, 69(17).
5. Ballisat, L., De Sio, C., Beck, L., Guatelli, S., Sakata, D., Shi, Y., Duan, J., Velthuis, J., & Rosenfeld, A. (2024). Dose and DNA damage modelling of diffusing alpha-emitters radiation therapy using Geant4. Phys Med, 121, 103367.
6. Lielkajis, M. D., Rosenfeld, A. B., Guatelli, S., Metcalfe, P. E., Poder, J., Bucci, J. A., & Cutajar, D. L. (2026). Diffusing alpha-emitters radiation therapy dosimetry: Geant4 multi-stage Monte Carlo modeling of sources. Med Phys, 53(6), e70494.