

Tree-Based Beam Angle Optimisation for Proton Therapy: An Automated Planning Tool

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Abstract

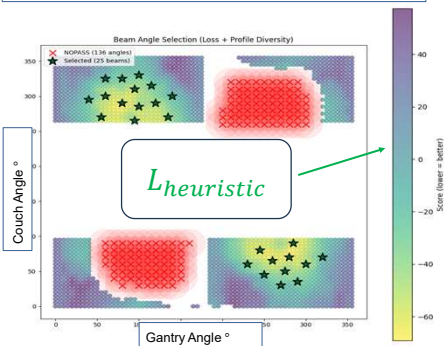
We developed an automated pipeline for proton therapy beam angle selection utilising heuristic pruning. This method consistently produced plans that performed similarly to clinical baselines even under robustness, validated on 5 pituitary patients.

Introduction

Previous Methods: Existing strategies for BAO typically split into gradient descent or mathematical programming methods to find locally optimal multi-field solutions [1], or lightweight heuristic beam-scoring approaches [2].

Our Method: We frame BAO as a pruned tree search: lightweight heuristic scoring and simulated annealing prune the branches of the full angular space. Surviving leaves are few enough to evaluate exhaustively under range and setup uncertainty.

Stochastic Multicriteria Acceptability Analysis (SMAA) prunes the leaves themselves, retaining only plans that remain optimal as clinical priorities shift, yielding a robust set of distinct trade-offs.



Method - Multi-Objective Beam Angle Optimisation (BAO)

1. Candidate Pruning (Heuristic Loss)

- For (Gantry, Couch) angle pair: CT array rotated to Beam's Eye View (BEV); CTV expanded margins.
- Per-angle metrics: BEV-OAR overlap, Bragg-Kleeman energy deposition, HU heterogeneity.

$$L_{heuristic,l} = \left(\sum_{j=1}^N z_{jl} \times w_j \right) + HU_{inter,l} + HU_{intra,l}$$

where, for candidate angle l :

- z_{jl} — z-scored energy-weighted BEV _{l} - OAR _{j} overlap across all angles
- w_j — clinical importance weight of OAR _{j}
- $HU_{intra,l}$ — within-slice HU σ averaged over all slices
- $HU_{inter,l}$ — slice-to-slice HU jumps

2. Top-K Selection with Geometric Diversity

- Simulated Annealing (SA) selects angles balancing low loss against a beam-separation penalty.
- Robust to OAR weight choice: Chamfer distance $6.1 \pm 1^\circ$ under weight perturbation vs. $4.5 \pm 0.7^\circ$ across independent runs.

3. Dose Calculation (FRED Monte Carlo [3])

- Compute dose for best K (K=25) performing plans.
- FRED MC builds pencil-beam influence matrix for re-weighting.

4. Naïve Plan Optimisation of Expanded CTV

- Simulate setup shifts $\pm 3\text{mm} \pm 3.5\%$ HU-to-RSP range uncertainty.
- Per-angle L-BFGS optimisation (coverage, shell dose, spot sparsity, hot/cold spots).

5. Exhaustive Search Over Beam Combinations & Stochastic MC

- Plan loss inspired by changing clinical goals.

$$L_j = \sum_{i \in \text{active}} w_i \times sev_{ij} \times (1 + g_{ij} \times \text{penalty}_{ij}) + w_{bp} \times z_{bp,j}$$

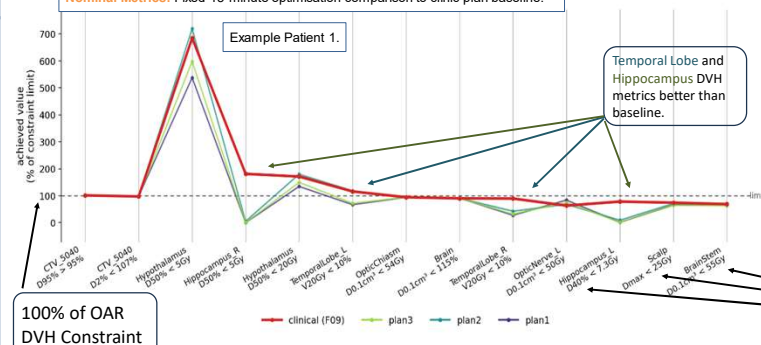
summed over active OARs:

- w_i — OAR clinical weight
- sev_{ij} — how poor plan j is for OAR i vs. the cohort mean (DVH / limit, cohort-normalised)
- $\text{penalty}_{ij} = \max(0, e^{w_i \times z_{ij}} - 1)$ — exponential penalty on the robustness-aware cohort Z-score Z_{ij}
- Z_{ij} — Z-score likelihood-weighted combination of nominal and worst-case DVH metric
- g_{ij} — sigmoid gate that limits the penalty for plans still under the clinical limit
- $z_{bp,j}$ — beam-path heterogeneity metric (z-scored) and its weight w_{bp}

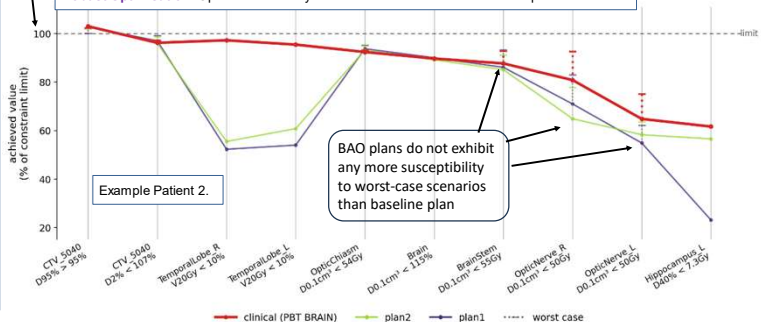
- SMAA [4]:** 10,000 rollouts across 5 clinical perspectives (Cautious→Bullish).

- Top frequency rated plans are selected; visualise selected plans with central weight vectors.

Nominal Metrics: Fixed 15-minute optimisation comparison to clinic plan baseline.



Robust Optimisation: Optimised to satisfy worst-case scenarios to test inherent plan robustness.



Testing Pipeline

Objective: Evaluate the clinical viability of an automated BAO pipeline against clinical baseline plans.

Optimisation Bias: Prioritised sparing of the Temporal Lobe and Hippocampus.

OAR DVH Constraints

Conclusion

We demonstrate an automated BAO pipeline that successfully produces beam combinations that quality match clinically delivered plans for pituitary cases.

We aim to further test the pipeline on other sites. Our end term aim it to produce a robust, highly efficient, and clinically ready solution for automated proton treatment planning.

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