

Determining Alpha DaRT (^{224}Ra) source requirements for ellipsoidal tumors using a 3D close packing algorithm

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INTRODUCTION

Diffusing alpha-emitters Radiation Therapy (DaRT) requires accurate placement of radioactive sources to ensure adequate coverage of the gross tumor volume (GTV) while minimizing the total number and length of implanted sources [1]. In this work, we present a simple and flexible algorithm to estimate the number and combination of Alpha DaRT sources required to cover spherical and ellipsoidal tumors, based solely on tumor geometry and seed lengths.

METHODS AND MATERIALS

To model the spatial distribution of Alpha DaRT sources in spherical or ellipsoidal tumors, we first discretized either the great circle of a spherical tumor or the central elliptical cross-sectional plane of an ellipsoidal tumor by covering it with circles of radius r_0 , which can be chosen arbitrarily. In this work, we set $r_0 = 2$ mm, corresponding to a lattice spacing of 4 mm as used by Heger *et al.* [1] and Epstein *et al.* [2]. Notably, r_0 can be defined according to the isodose of interest (e.g., $r_0 = 2.21$ mm for the 20 Gy alpha particle physical dose, or $r_0 = 2.23$ mm for the 100 Gy-Eq total equivalent dose of all radiation components). Cylinders of radius r_0 (representing a therapeutic dose surrounding Alpha DaRT sources) were positioned at the centers of these circles (on the base of the hemisphere or hemi-ellipsoid) and oriented perpendicular to the base plane. For each cylinder, the maximum height from the base plane to the tumor surface was calculated. Let h denote the height of a given cylinder. The source configuration $S(h)$ assigned to each cylinder was defined as:

$$S(h) = \begin{cases} \{10\} & \text{If } h \leq 10 \text{ mm,} \\ \{20\} & \text{If } 10 < h \leq 20 \text{ mm} \\ \{10, 20\} & \text{If } 20 < h \leq 30 \text{ mm} \\ \vdots & \end{cases}$$

where the values in brackets indicate the source lengths in millimeters. This algorithm ensures complete geometric coverage of spherical and ellipsoidal tumors while minimizing the total implanted source length required.

RESULTS

A Python code was developed to calculate the required number of Alpha DaRT sources for ellipsoidal tumors with arbitrary dimensions and to determine the optimal combination of 10 mm and 20 mm sources. Figure 1 illustrates four representative source arrangements for a tumor volume of 7.54 cc, including a spherical tumor model (diameter: 24.32 mm) and an ellipsoidal tumor model (dimensions: 20 mm $\times$ 24 mm $\times$ 30 mm). Three representative treatment configurations and their associated cylindrical arrangements are shown. The top-view projection at $z = 0$ mm corresponds to the central elliptical cross-section of the ellipsoidal tumor. Table 1 summarizes the number of 10 mm and 20 mm Alpha DaRT sources required for the spherical tumor, the ellipsoidal tumor with the same volume, and two additional configurations of the ellipsoidal tumor obtained by permuting its principal axes. The total implanted source length is also reported for each configuration.

Table 1: Number of 10 mm and 20 mm Alpha DaRT sources required to cover spherical and ellipsoidal tumors under different geometric configurations.

Tumor size (mm)	10 mm sources	20 mm sources	Total source length (mm)
20×24×30	21	23	670
30×20×24	19	23	650
30×24×20	8	35	780
24.32×24.32×24.32	13	31	750

DISCUSSION

This study presents a simple and flexible algorithm to estimate the optimal number and combination of Alpha DaRT sources required for spherical and ellipsoidal tumors. The results show that tumor geometry and implantation orientation significantly influence key treatment parameters, particularly the total required source length. Accordingly, a Python-based computational tool was developed to rapidly suggest appropriate combinations of 10 mm and 20 mm sources for efficient pre-treatment planning and source ordering. The algorithm models cylindrical elements placed perpendicular to the base plane and calculates their maximum height to the tumor surface to ensure adequate geometric coverage. These findings emphasize the importance of three-dimensional modeling and spatial orientation to optimize resource utilization and ensure effective treatment planning.

CONCLUSIONS

The results demonstrate that tumor shape and implantation orientation significantly influence the total required source length. The proposed algorithm is directly applicable for rapid estimations in simple ellipsoidal geometries, aiding in initial case assessment and source ordering.

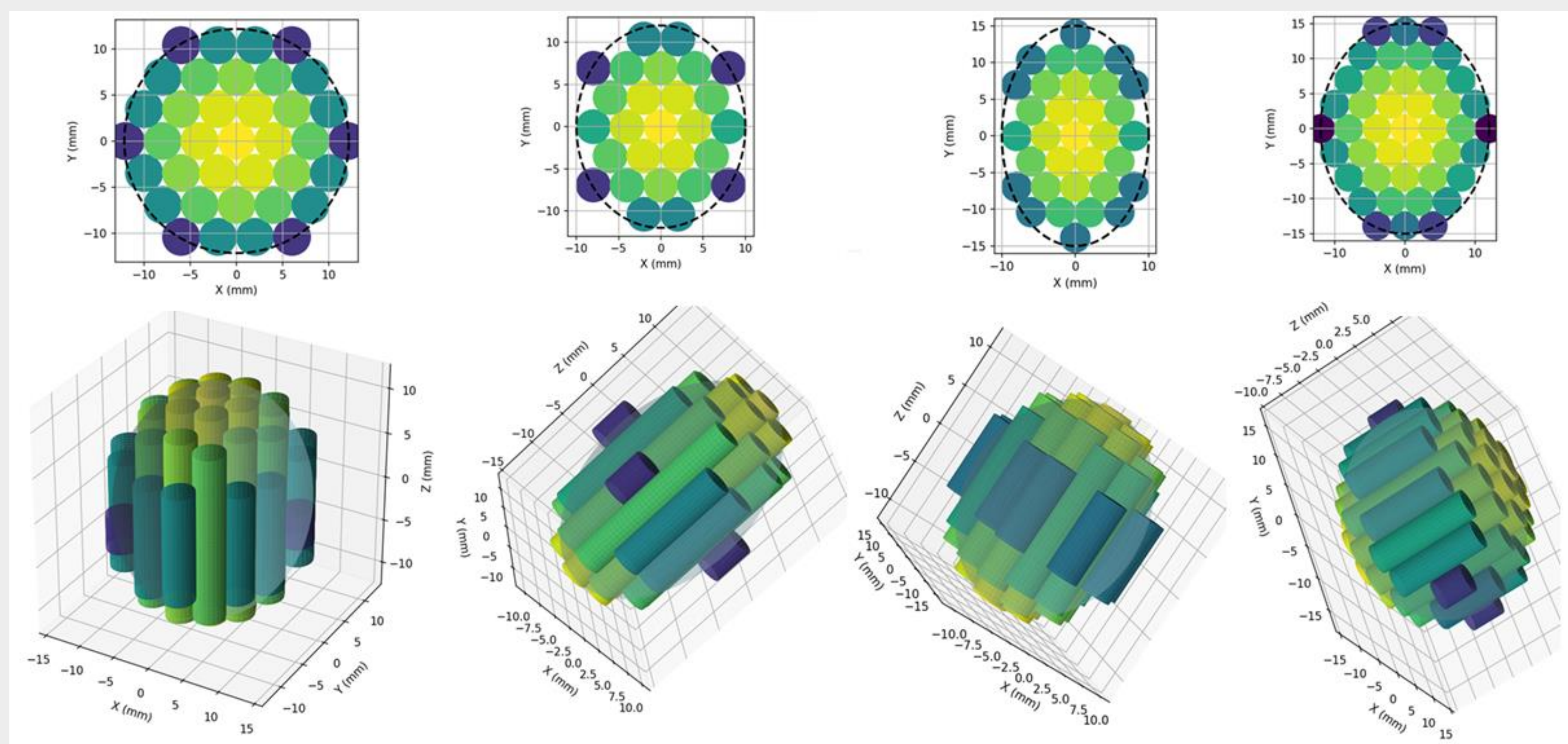


Figure 1: The spherical tumor model for a 24.3 mm diameter tumor and an ellipsoidal tumor model (20 $\times$ 24 $\times$ 30 mm diameters), illustrating three orientations of implantation for the ellipsoid and their corresponding source configurations.

REFERENCES

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