Physikalisch-Technische Bundesanstalt
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Type-B uncertainty evaluation of Monte-Carlo dose calculations for a clinical linac

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Abstract

Purpose: To determine the Type-B uncertainty of dose calculations using a Monte-Carlo model of a clinical linac and to quantify the contributions of different sources of uncertainty.

Methods: A Monte Carlo model of the Elekta Synergy LINAC, operated at the German National Metrology Institute producing a 6 MV filtered photon beam, was commissioned, achieving 100 % agreement using a (1 mm, 1 %) gamma passing rate along the percentage depth dose (PDD) curve and both transverse dose profiles for multiple square field sizes. Subsequently, following the procedure of the JCGM *Guide to the Expression of Uncertainty in Measurement* (GUM)[1], the combined standard uncertainty was determined by carrying out extensive Monte Carlo simulations of dose distributions in a water phantom using *EGSnrc*[2], while varying individual parameters representing distinct sources of uncertainty.

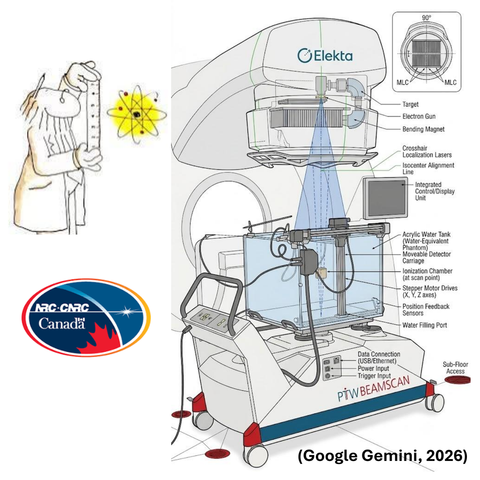
Results: The spatially resolved standard uncertainty of the relative dose distributions was determined for three different square field sizes. The largest uncertainty contribution near the surface along the PDD was found to be below 1 % and arises from uncertainties in the primary electron properties. Beyond the depth of dose maximum, uncertainties in photon interaction cross sections contributed most to the total uncertainty (< 1 % within the observed depth range of 25 cm. Along the transverse dose profiles, significant uncertainties arose from multiple sources (in-plane < 8 %, cross-plane < 12 %, excluding positioning uncertainties). The product of the spatially resolved combined standard uncertainty and the relative dose value at each respective position resulted in deviations below 1 % across the entire PDD and 4 % across the transverse dose profiles.

Conclusions: Although Monte Carlo simulations are widely regarded as the ground truth for dose calculations, they exhibit uncertainties that cannot be assessed using statistical methods alone. For the LINAC model discussed in this work, a conservative uncertainty budget was established that yields small overall uncertainties.

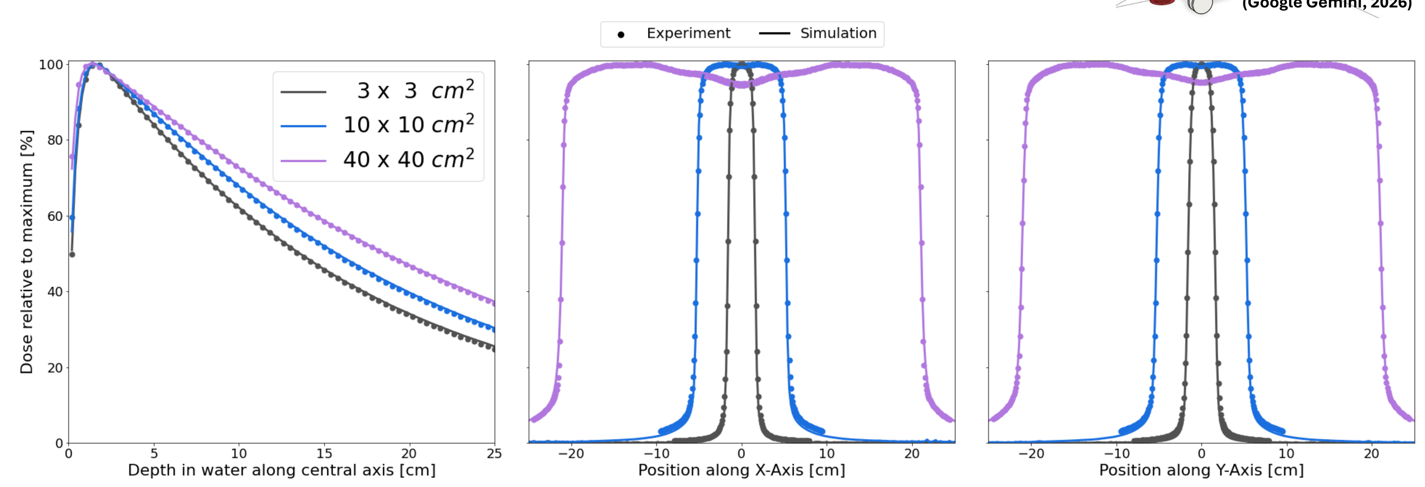
Introduction

Following the commissioning of a clinical linac, used to investigate fundamental dosimetric questions at the German National Metrology Institute (PTB), a Monte Carlo model was established by comparing simulated and measured dose profiles in a water phantom.

The purpose of this work was to study the effect of sources of uncertainty within the simulation and to investigate how they might affect derived quantities such as radiation quality correction factors.



(Google Gemini, 2026)



Methods

The relevant components within the linac treatment head were modeled using confidential, vendor-specific information, within the user code *BEAMNrc* distributed as a part of the Monte-Carlo software toolkit *EGSnrc*[2] and optimized by iteratively adjusting the primary electrons properties impinging on the bremsstrahlungs target such that both transverse dose profiles in multiple depths match and percentage depth dose profiles agree with experimental data for multiple square field sizes ranging from 3x3 cm² to 40x40 cm². Dose profiles in a water phantom were simulated using *egs_chamber*. The transport cutoff energies were set to 0.521 MeV globally but range rejection within the water phantom to the voxels of the dose profiles was used (*ESAVE* 1 MeV).

The individual contributions for all sources of uncertainty x_i were calculated by approximating the sensitivity coefficients c_i using the numerical derivative as proposed by Carlan et al. (2008) and reproduced by others[3,4]. Consequently, percentage depth dose (PDD) and two-dimensional dose profiles at a depth of 10 cm were calculated for each of the 22 sources of uncertainty and three field sizes. The Monte Carlo intrinsic statistical noise was accounted for by calculating the Euclidean distance to the uncertainty contribution.

$$u_c(\mathcal{Y}) = \sqrt{\sum_{i=1}^{N}\left[\frac{\partial \mathcal{F}}{\partial x_i}\, u(x_i)\right]^2} = \sqrt{\sum_{i=1}^{N}\left[c_i\, u(x_i)\right]^2} \qquad c_i = \sqrt{{c_i^{\dagger}}^2 + \sigma_{stat}^2}$$

$$c_i^{\dagger} = \frac{\partial \mathcal{F}(x_1, ..., x_N)}{\partial x_i} = \frac{\mathcal{F}(x_1, ..., x_i, +\Delta x_i, ..., x_N) - \mathcal{F}(x_1, ..., x_i, ..., x_N)}{\Delta x_i}$$

Previous studies[5] demonstrated experimentally that the electron focal spot position on the bremsstrahlungs target can deviate from the symmetry axis. To study how a translation and rotation of the two-dimensionally distributed electrons affects dose profiles, a new particle source was written in Fortran and integrated into BEAMnrc.

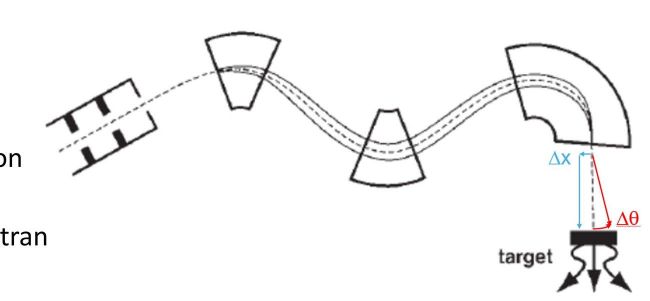


Table 1 shows the uncertainty budget used to determine the combined standard uncertainty and demonstrates the effect of each source of uncertainty on the voxel corresponding to the depth at which the beam quality specifier *%dd(10)*$_x$ for high-energy photon beams, as defined by TG-51[6], is determined.

Source of uncertainty x_i	Change Δx_i	$u(x_i)$	Distribution of x_i	$\lvert c_i u(x_i)\rvert$
Target Thickness	100 µm	50 µm	Gaussian	0.076 %
Target Composition	2% $^{184}_{74}$W	2% $^{184}_{74}$W	Gaussian	0.080 %
Target Density	0.2 $g\,cm^{-3}$	0.1 $g\,cm^{-3}$	Gaussian	0.074 %
Flattening Filter Density	0.3 $g\,cm^{-3}$	0.15 $g\,cm^{-3}$	Gaussian	0.070 %
Leaf Density	0.3 $g\,cm^{-3}$	0.3 $g\,cm^{-3}$	Gaussian	0.114 %
Leaf Positioning	2 mm	1 mm	Rectangular	0.078 %
Energy	3 %	1 %	Gaussian	0.125 %
Spot Size X	20 %	5 %	Gaussian	0.070 %
Spot Size Y	20 %	5 %	Gaussian	0.076 %
Mean Angular Deviation	1.25 °	0.625 °	Gaussian	0.110 %
Translation X	500 µm	250 µm	Gaussian	0.074 %
Translation Y	500 µm	250 µm	Gaussian	0.086 %
Rotation X	0.1 °	0.1 °	Gaussian	0.127 %
Rotation Y	0.1 °	0.1 °	Gaussian	0.090 %
Rayleigh	2 % (across all media)	2 % (across all media)	Gaussian	0.101 %
Photoelectric Effect	2 % (across all media)	2 % (across all media)	Gaussian	0.087 %
Compton scattering	2 % (across all media)	2 % (across all media)	Gaussian	0.267 %
Pair production	2 % (across all media)	2 % (across all media)	Gaussian	0.086 %
Electron Impact Ionisation	2 % (across all media)	2 % (across all media)	Gaussian	0.097 %
SSD	3 mm	2 mm	Rectangular	0.104 %
X-Position	3 mm	2 mm	Rectangular	0.077 %
Y-Position	3 mm	2 mm	Rectangular	0.079 %

Combined standard uncertainty $u_c = \sqrt{\sum_i [c_i u(x_i)]^2} = 0.992\%$ (k = 2)

Results

The graphs on the right show the spatially resolved standard uncertainty along the percentage depth dose curve (left) and the two-dimensional dose profile at a water depth of 10 cm for 3x3 cm², 10x10 cm², and 40x40 cm² grouped by their origin (different rows).

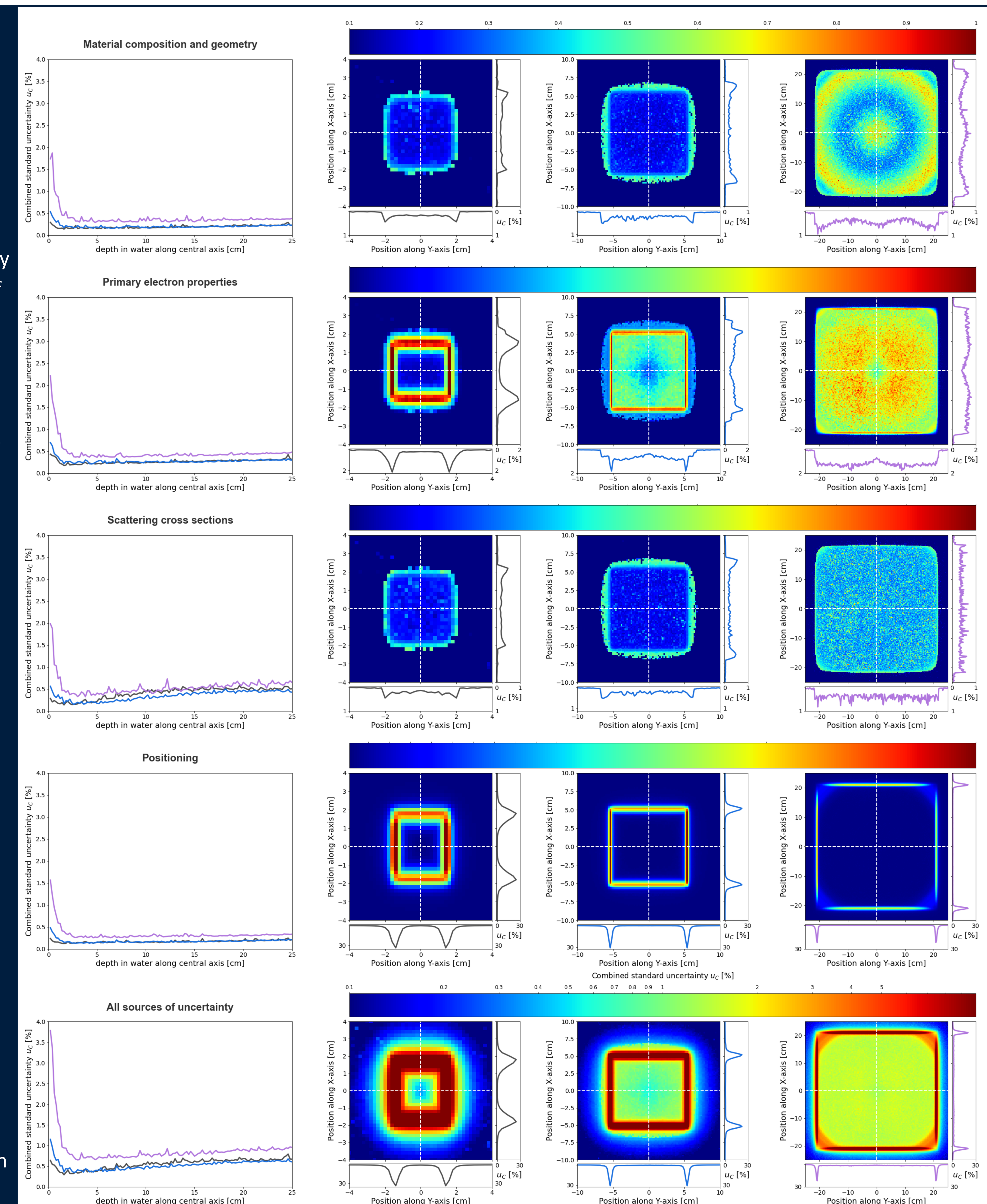
Consequently, the first row shows the combined standard uncertainty for quantities linked to uncertainties in the material and geometry of the treatment head (first block separated by a line in the uncertainty budget). The uncertainties along the percentage depth dose curve remain within 0.5 % after the depth-dose maximum and reach up to 1.75 % for the large 40x40 cm² field close to the surface. The transverse dose profiles show a small uncertainty contribution at the penumbra for smaller fields due to leaf density uncertainty, while larger fields exhibit an additional uncertainty linked to the density of the flattening filter, which has less effect on smaller fields. These transverse uncertainties remain within 1 %.

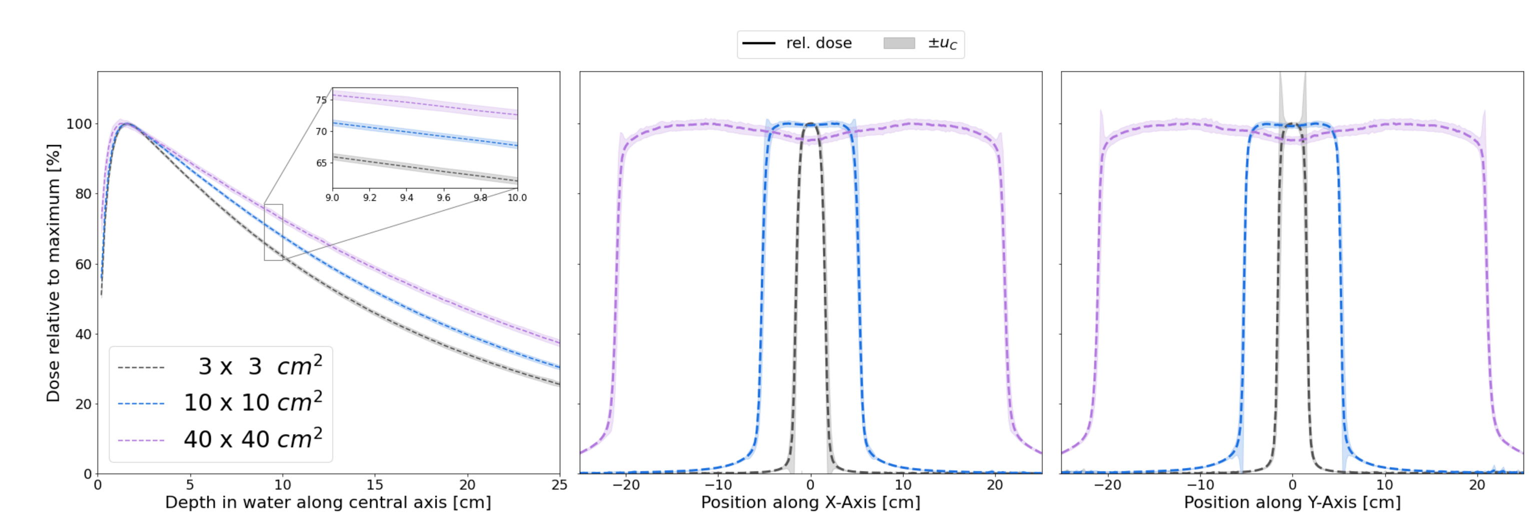
The second row corresponds to the uncertainty in the primary electron properties (second block of quantities in the uncertainty budget). While the penumbra region is largely affected due to changes in the electron focal spot size for all field sizes (especially small fields), the large field is strongly affected in the plateau region by changes in angular deviations and energy. The uncertainty along the PDDs is most affected by changes in energy

The third row indicates the uncertainties of the cross-section, which affect the PDD mostly due to changes in the Compton scattering cross-section (at 6 MV, the most probable interaction in water). Electron impact ionization effects the surface region as well. The effects on the transverse dose profile are small.

The final group of uncertainties is linked to the positioning uncertainties of the 2 mm³ large scoring regions within the water phantom. In pure simulation studies, this uncertainty group is obsolete. Nevertheless, it is a quantity that cannot be ruled out when carrying out experiments and thus affects the comparison of simulations and experimental data, which is why it's included in this investigation. The uncertainties in the penumbra of the transverse dose profiles represent the largest contributions of all quantities and reach up to 30 %. The effect decreases with field size; it should also be noted that the effect on the PDD is largest for large fields as well.

Finally, the combined standard uncertainty is shown as the 4th row and below the normalized dose profiles and PDD are shown with the k=1 confidence interval on the right.





Conclusion

With this particular approach to numerically determine the combined standard uncertainty for Monte Carlo dose calculations, we were able to quantify the contribution of different sources of uncertainties that might occur in simulating a clinical linac. Our findings demonstrate that small fields exhibit large uncertainties in the penumbra region of the transverse dose profiles, while their plateau region is less affected. Conversely, larger fields are affected in the plateau regions as well, alongside significant uncertainties in the penumbra. Additionally, the uncertainty along the PDD is largest for large fields. Over the observed depth of 25 cm, the combined standard uncertainty remains within 1 % after the dose maximum for all field sizes.

It should be noted that generalizing these findings and transferring them to different machines or energies should only be done after due consideration, as changing for example the energy from 6 MV to 15 MV will alter the probability of each interaction, leading to increased pair production over Compton scattering in water. Aside from this, uncertainties in the penumbra strongly depend on the collimator shape and are vendor/machine-specific.

Considering the large number of quantities investigated and the overall very conservative estimation of different uncertainties $u(x_i)$ these results are very satisfying as they indicate the robustness of Monte Carlo dose calculations for a clinical linac.

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