Numerical evaluation of hemodynamic-derived quantities using the meshless method: An application to Wall Shear Stress

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ABSTRACT

Purpose : Accurate evaluation of near-wall hemodynamic metrics, i.e. wall shear stress (WSS), time-averaged wall shear stress (TAWSS), and oscillatory shear index (OSI), is critical for understanding vascular disease progression. These indices depend on spatial derivatives of velocity and pressure fields, which are challenging to compute robustly on complex arterial geometries. We describe a new meshless strong-form method based on Discretization-Correction Particle Strength Exchange (DC PSE) for accurate computation of spatial derivatives of unknown field functions in arterial blood flow simulations.

Methods: The proposed method operates directly on scattered point clouds without the need for a predefined connectivity (mesh). Spatial derivatives are approximated using PSE operators augmented with discretization correction terms enforcing polynomial consistency and significantly reducing truncation errors on irregular nodal distributions. The method is formulated in strong form and integrated within an incompressible Navier-Stokes solver for pulsatile blood flow. Near-wall derivative reconstruction is handled consistently using one-sided corrected operators, enabling direct evaluation of velocity gradients at the arterial wall. Hemodynamic indices, instantaneous WSS, TAWSS, and OSI, are computed from the reconstructed gradients over the cardiac cycle. The method is applied on a benchmark flow case, and on an ascending and descending aorta.

Results: Numerical experiments on idealized and patient-specific arterial geometries demonstrate that DC PSE achieves high-order accuracy for first- and second-order spatial derivatives on highly irregular point sets. DC PSE produces smooth and physically consistent WSS fields, with improved stability and reduced numerical noise compared to conventional meshless collocation and uncorrected PSE approaches.

Conclusion: The proposed DC PSE meshless strong-form method provides an accurate and robust framework for computing spatial derivatives of unknown fields in arterial blood flow simulations. Its ability to reliably evaluate hemodynamically important derived quantities such as WSS-based hemodynamic indices on complex geometries makes it a promising tool for computational hemodynamics and cardiovascular research.

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INTRODUCTION

Accurate evaluation of near-wall hemodynamic metrics, such as wall shear stress (WSS), time-averaged wall shear stress (TAWSS), and oscillatory shear index (OSI), is critical for understanding vascular disease progression. These indices depend on spatial derivatives of velocity and pressure fields, which are challenging to compute robustly on complex arterial geometries.

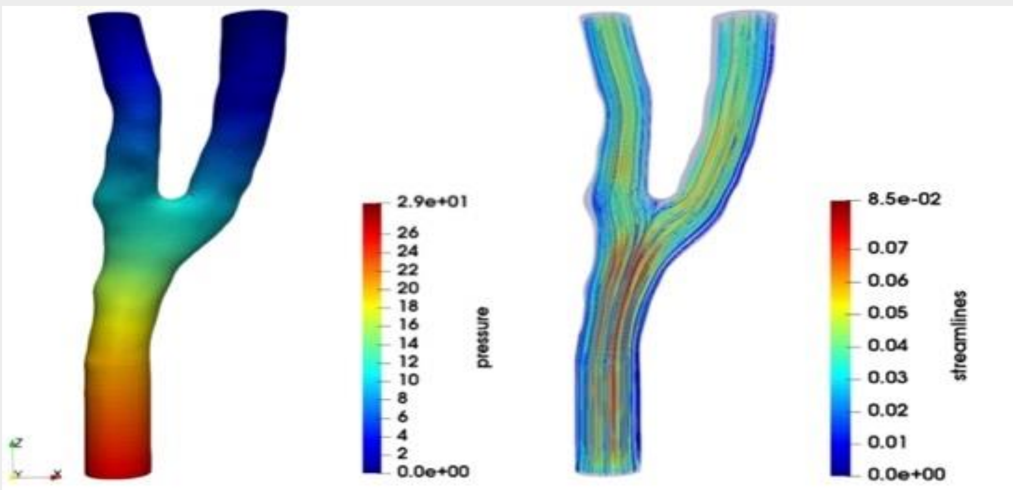
In meshless computational frameworks, the accurate estimation of these quantities is particularly demanding because derivative operators are constructed directly from scattered point clouds rather than from predefined mesh connectivity. Advanced discretization techniques, such as the Discretization-Correction Particle Strength Exchange (DCPSE) [1,2], provide high-order approximations of velocity gradients while preserving geometric flexibility. Their ability to represent complex vascular anatomies without the burden of mesh generation makes them attractive for patient-specific hemodynamic simulations.

Reliable prediction of WSS-related biomarkers requires not only accurate spatial discretization but also stable temporal integration and robust treatment of boundary conditions. Numerical errors in the vicinity of vessel walls can significantly affect the computed shear stress distributions and consequently alter the identification of regions susceptible to atherosclerosis, aneurysm growth, or restenosis.

The present work focuses specifically on the post-processing phase of the CFD pipeline, investigating how differences in WSS evaluated via the finite element method (FEM) arise even when the geometry, boundary conditions, viscosity, and so forth, are held fixed.

METHODS AND MATERIALS

The proposed method operates directly on scattered point clouds without the need for a predefined connectivity (mesh). Spatial derivatives are approximated using PSE operators augmented with discretization correction terms enforcing polynomial consistency and significantly reducing truncation errors on irregular nodal distributions. The method is formulated in strong form and integrated within an incompressible Navier-Stokes solver for pulsatile blood flow. Near-wall derivative reconstruction is handled consistently using one-sided corrected operators, enabling direct evaluation of velocity gradients at the arterial wall. Hemodynamic indices, instantaneous WSS, TAWSS, and OSI, are computed from the reconstructed gradients over the cardiac cycle. The method is applied on a benchmark flow case, and on an ascending and descending aorta.



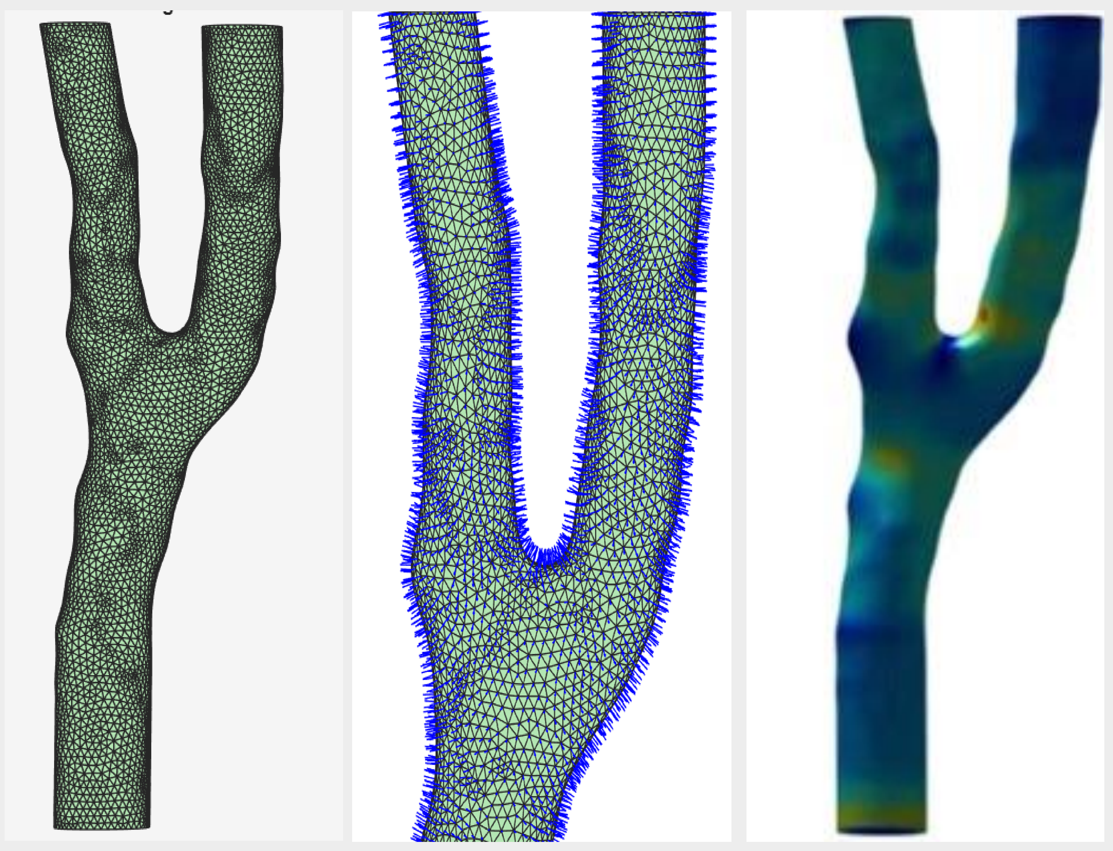
RESULTS

WSS is defined as the tangential component of the traction force per unit area exerted by a flowing fluid on a boundary. It is computed as the magnitude of the surface traction vector $\boldsymbol{t}_s=\boldsymbol{t}-(\boldsymbol{t}.\boldsymbol{n})\boldsymbol{n}$, where $\boldsymbol{n}$ is the unit normal vector pointing outward, and $\boldsymbol{t}=2\mu\boldsymbol{\varepsilon}$ is the shear stress, defined based on the fluid's viscosity μ and the strain-rate tensor $\boldsymbol{\varepsilon}$. However, it is most commonly treated as a vectorial quantity, representing both the magnitude and direction of the tangential force. In the SI unit system, WSS is measured in pascals (Pa).

The continuously differentiable velocity vector field $\boldsymbol{u}(\boldsymbol{x},t)$, describes the blood flow at a point in the domain of interest. The velocity field is computed using a well-established mesh-based flow solver, using finite volume (FV) or finite element (FE) method. To compute WSS, we consider a discretization of the domain of interest using a point cloud (generated using the vertices of the elements used to discretize the flow domain). The nodal subset with points on the surface of the flow domain will be used for the calculation of WSS. Using indicial notation for the components of $\boldsymbol{u}$, the strain-rate tensor components are defined by $\varepsilon ij=1/2(ui,j+uj,i)$, where $ui,j=\partial ui/\partial xj$. The terms $\partial ui/\partial xj$ of the strain-rate tensor $\boldsymbol{\varepsilon}$ are computed on the surface points using the DC PSE method as $\partial ui/\partial xj= \Phi,j\ ui$, with Φ,j being the jth spatial derivative computed using DC PSE, and ui being a column vector of the ith velocity vector components.

<u>Poiseuille flow</u>: We demonstrate the accuracy of the proposed IB method by considering the Poiseuille flow in a straight cylindrical tube. The tube has length L = 10R and radius R = 0.005 m, The driving force of the flow is the pressure difference applied to the inlet and outlet. In our simulations, the density of the fluid was set to ρ = 1, 050 kg m−3, and the dynamic viscosity to μ = 0.00345 N s⁄m2. The exact solution for the velocity field components is uy = uz = 0 and ux (r) =Umax(1 −(rR)^2), with Umax = − 4$R^2\mu$dp/dz being the maximum velocity. By setting the pressure difference to dp/dz = 1 L (applying $p$inlet = 1 and $p$outlet = 0) the Reynolds number becomes Re = ρUmax$L$ μ ≈ 551, while for $p$inlet = 2 Reynolds number becomes Re = ρUmax$L$ μ ≈ 1, 102.

<u>Bifurcation</u>: We discretize the flow domain using tetrahedral elements. The point cloud consists of the vertices of the tetrahedral element. We apply the pulsatile velocity waveform (we use a parabolic velocity profile since the Womersley number is small), and zero pressure boundary conditions at the two outlets. At the remaining walls, we apply no-slip boundary conditions. We employ the Newtonian model for blood flow, with dynamic viscosity of μ = 0.00345 $Pa\cdot s$ and density of ρ = 1, 056 kg/m3.



DISCUSSION

We demonstrate the feasibility of computing wall shear stress (WSS) directly on vascular point cloud representations using the Discretization Corrected Particle Strength Exchange (DCPSE) method. Accurate estimation of WSS is a critical requirement in cardiovascular biomechanics because it provides insight into endothelial mechano-transduction, vascular remodeling, aneurysm progression, and atherosclerotic plaque development.

A major advantage of the proposed approach is the ability to compute spatial derivatives without the need for a volumetric mesh. Conventional finite element or finite volume formulations require high-quality boundary-fitted meshes and significant preprocessing effort, particularly for patient-specific vascular geometries exhibiting complex anatomical features. In contrast, the DCPSE framework operates directly on scattered point clouds, thereby reducing meshing requirements and enabling greater flexibility in handling irregular and highly curved vascular surfaces.

The point-cloud-based approach provides a direct link between image-derived vascular geometries and hemodynamic analysis. Since medical imaging techniques naturally generate surface point representations, the proposed framework minimizes the need for intermediate geometric processing steps, such as high-quality mesh generation and optimization. This characteristic has the potential to facilitate automated and efficient patient-specific simulations, supporting the development of computational tools for rapid assessment of vascular hemodynamics and biomechanical risk indicators.

The DCPSE formulation incorporates correction terms that improve the consistency of differential operator approximations, enabling more reliable evaluation of velocity gradients and, consequently, more accurate estimation of wall shear stress.

CONCLUSIONS

The results indicate that the DCPSE discretization provides sufficiently smooth and stable velocity gradient estimates near the vessel wall, leading to physically meaningful WSS distributions. The correction procedure incorporated into the DCPSE formulation improves the consistency and accuracy of differential operator approximations, which is particularly important in regions where point distributions are non-uniform or locally distorted. often exhibit varying spatial resolution

The proposed meshless strong-form method provides an accurate and robust framework for computing spatial derivatives of unknown fields in arterial blood flow simulations. Medical imaging modalities such as CT and MRI inherently generate point-based geometric information. Future work will focus on high-order adaptive point cloud refinement strategies.

REFERENCES

1. B. Schrader, S. Reboux, and I. F. Sbalzarini, "Discretization Correction of General Integral PSE Operators for Particle Methods," Journal of Computational Physics 229, no. 11 (2010): 4159-4182.
2. G.C. Bourantas, B.L. Cheeseman, R. Ramaswamy, I.F. Sbalzarini, Using DC PSE operator discretization in Eulerian meshless collocation methods improves their robustness in complex geometries, Comput. Fluids 136 (2016) 285-300.