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A Statistical Framework for Modeling Paralyzable Pulse Pile-up Effects in Photon-Counting X-Ray Detectors

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ABSTRACT

High X-ray fluxes in photon-counting CT (PCCT) cause pulse pile-up effects (PPEs), which may degrade imaging performance. Classical models are limited in characterizing the statistical properties of paralyzable pulse pile-up mechanisms across multi-energy thresholds.

In this work, we propose a new statistical framework to model paralyzable PPEs. This model explicitly captures the temporal correlations and statistical noise characteristics induced by pulse pile-up.

Validated against Monte Carlo simulations with under 5% deviation, this computationally efficient approach is promising to provides a robust theoretical foundation for precise PCCT performance analysis and the development of advanced noise-reduction algorithms.

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INTRODUCTION

While the deterministic effects of PPEs and the statistical behaviors of non-paralyzable models have been widely studied[1-3], the statistical properties associated with paralyzable mechanisms remain largely underexplored. Classical analytical approaches for paralyzable PPEs[4] are limited to provide a precise characterization of statistical properties—specifically variance and covariance—across multi-energy thresholds.

In this work, we propose a new statistical framework to model paralyzable PPEs. It is grounded in the concept of the baseline probability density distribution. By deriving the joint characteristic functions of baseline states and applying the Campbell-Bartlett theorem, we establish analytical expressions for the mean count rates, variances, and the covariance between arbitrary energy thresholds.

METHODS

A. Stochastic Description of Pulse Pile-up

The analog signal generated by the detector is modeled as a stochastic process driven by a Poisson point process. Considering the coupling between the incident spectrum $S(E)$ and energy response $R(E,U)$, the probability density function $P_U(U)$ of the pulse height and the cumulative height spectrum $Q(U)$ are defined as follows:

$$P_U(U) = \int S(E)R(E,U)\mathrm{d}E\,.$$

$$Q(U) = \int_U^\infty P_U(U')\mathrm{d}U'\,.$$

The instantaneous baseline amplitude X_n of the detector output can be expressed as:

$$X_n = Y(t_n^-) = \sum_{m<n} U_m\, g(t_n - t_m).$$

Where $g(t)$ is the pulse kernel function used to describe the pulse shape.

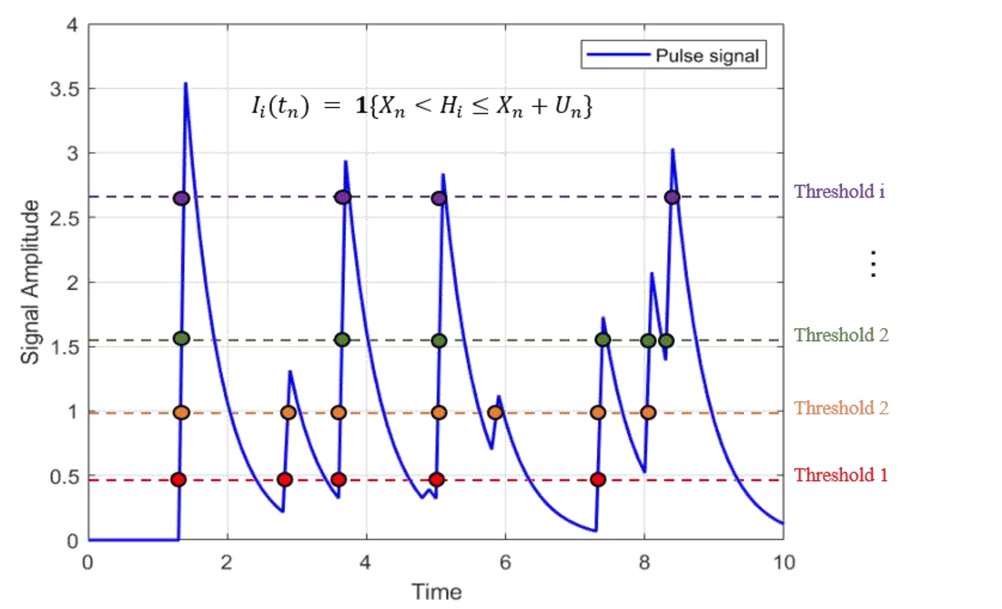

Fig 1. Up-crossing counting mode studied in this work.

B. Mean and Covariance of Threshold Counts

Let C_i denote the random count recorded at energy threshold H_i during a measurement interval T. The mean count is:

$$N_i = \mathbb{E}[C_i] = \lambda T(Q(U) * P_X(U))(H_i).$$

$$P_X(U) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}uU} \cdot \mathrm{e}^{-\int_0^\infty S(E)K(E,u)dE}\mathrm{d}u.$$

The expression for N_i is mathematically equivalent to the counting formula proposed by Roessl[5].

Applying the Campbell theorem[6] to the second-order statistics of the threshold-crossing events gives:

$$\boxed{Cov(C_i,C_j) = \lambda T\,\mathbb{E}[I_iI_j] + \lambda^2\int_{-T}^{T}(T-|\tau|)(\mathcal{P}_{ij}(\tau) - n_in_j)\mathrm{d}\tau\,.}$$

Here:

- *λ: incident photon arrival rate,*
- *$P_X(U)$: probability density of the baseline X*
- *I_i: $I_i(t_n) = \mathbf{1}_{\{X_n<H_i\le X_n+U_n\}}$, indicator function for an upward crossing of threshold U_i*
- *n_i: $n_i = \mathbb{E}[I_i]$, counting probability per incident photon at H_i,*
- *$\mathbb{E}[I_iI_j]$: probability that the same photon crosses both thresholds,*
- *τ: time separation between two photon arrivals,*
- *$\mathcal{P}_{ij}(\tau)$: ordered joint probability that two photons separated by trigger H_i and H_j, respectively.*

The first covariance term is the same-photon Poisson shot-noise contribution. The integral is the temporal-correlation contribution, arising because residual pulses and correlated baseline states make the two threshold-crossing events dependent. The evaluation of $\mathcal{P}_{ij}(\tau)$ relies on the joint statistics of the baseline states, as described in the following subsection.

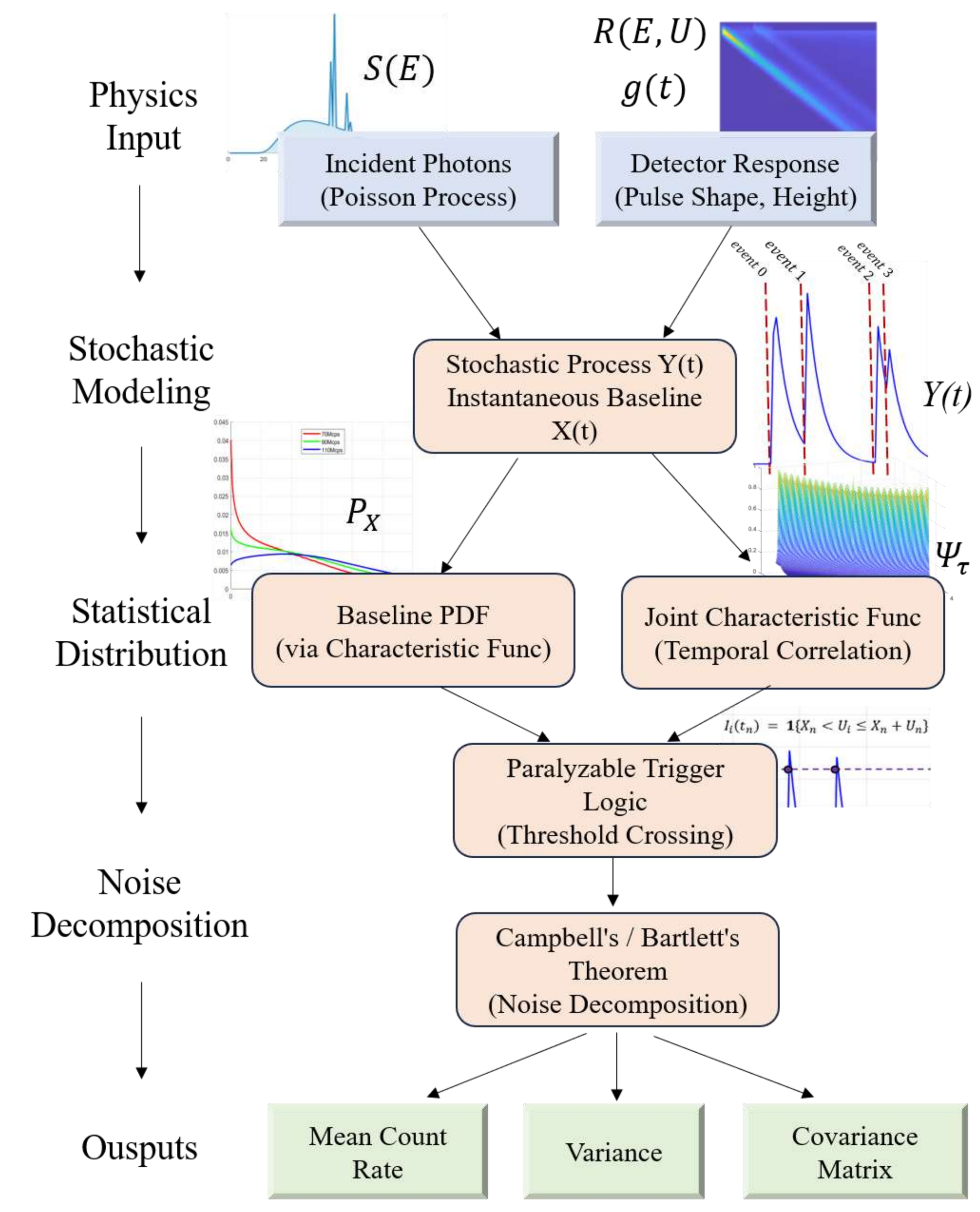

Fig 2. Flow of the Proposed Statistical Framework.

C. Joint Statistical Properties

$\mathcal{P}_{ij}(\tau)$ can be obtained by inverse Fourier transforming the joint characteristic function $\Psi_\tau(u_1,u_2)$ into the joint baseline density and averaging the joint threshold-crossing probability:

$$\mathcal{P}_{ij}(\tau) = \langle \mathcal{F}_{2D}^{-1}\{\Psi_\tau(u_1,u_2)\}(x_0,x_\tau), \mathcal{T}_{ij,\tau}(x_0,x_\tau)\rangle.$$

Where $\mathcal{T}_{ij,\tau}$ denotes the conditional probability that the two photons trigger thresholds H_i and H_j, respectively. $\Psi_\tau(u_1,u_2)$ can be expressed as:

$$\Psi_\tau(u_1,u_2) = \mathbb{E}[e^{u_1X_0+u_2X_\tau}]$$
$$= exp\left(-\lambda\int_0^\infty S(E)K_{joint}(E;u_1,u_2;\tau)dE\right).$$
$$K_{joint}(E;u_1,u_2;\tau)$$
$$= \int_0^\infty (1 - R(E,u_1g(\tau) + u_2g(t+\tau)))\mathrm{d}t$$
$$+ \int_0^\tau (1 - R(E,u_2g(t)))\mathrm{d}t\,.$$

RESULTS

Validation against Monte Carlo simulations demonstrated that the proposed model accurately predicted the covariance matrix, with a maximum Relative Frobenius Deviation (RFD) of less than 5% across various count rates.

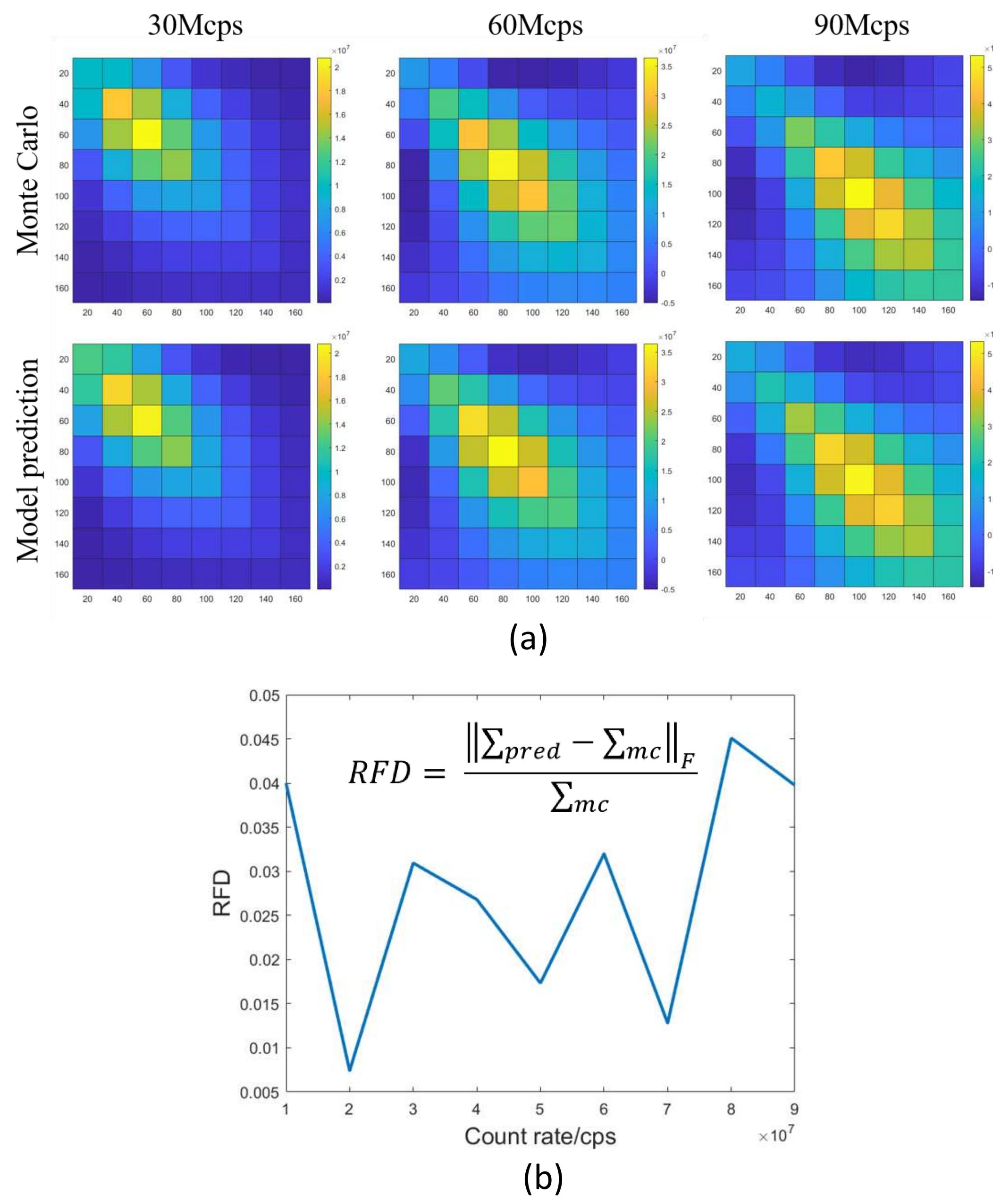

Fig 3. Validation against Monte Carlo simulations. (a) Covariance matrices of multi-threshold counts: Monte Carlo and model prediction. (b) RFD across various count rates.

CONCLUSION

We propose a statistical framework for paralyzable pulse pile-up in PCCT. It accurately characterizes multi-threshold count statistics and is expected to support precise PCCT imaging performance assessment, threshold optimization, and the development of statistically informed noise-reduction and reconstruction algorithms.

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